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Discussion

On "3-D inversion of gravity and magnetic data with depth resolution" (Maurizio Fedi and Antonio Rapolla, GEOPHYSICS, 64, 452-461).

We have read with great interest the paper entitled "3D inversion of gravity and magnetic data with depth resolution" by Fedi and Rapolla. We realize that many discussions we had with one of the authors had not resolved our differences and, therefore, we are writing this comment to document out concerns regarding the authors' conclusion that the upward-continued gravity or magnetic data contain extra information and provide depth resolution.

As is well known in potential-field theory, the fields produced by the subsurface source at any two nonintersecting observational surfaces are linearly related to each other and one contains no more information than does the other. Furthermore, the field on the observational surface above the source can be reproduced by an infinite number of equivalent sources below that surface.

As a consequence, gravity or magnetic data do not provide any information about the subsurface structure unless the source is assumed to have certain restrictive properties. No amount of upward continuation will create any new information and, consequently, the addition of upward-continued data will not resolve the depth distribution of the source. In practical applications, when the data are available in a small area and/or when the data are coursely sampled, then exact numerical continuation is not possible to perform. In such circumstances, independently-measured data at a different level will provide extra information that is not contained in the data at the original level. The maximum amount of additional information that any upper-level data can supply is that which is contained in the missing portion of the data map at the lower level.

Fedi and Rapolla provide two examples which they claim support their hypothesis that data along the vertical direction are needed to obtain depth resolution and, furthermore, that upward-continued data serve this purpose. As is shown below, both results arise because of a priori restrictions upon the model space used to carry out the inversion.

The first example (Figures 3 and 4 of their paper) compares the recovery of a buried susceptible block when only surface data are used, and when data at 10 different heights are used. When the surface data are inverted, the susceptibility maximum is at the surface, whereas when data at all 10 levels are inverted, the recovered susceptibility compares well with the true values. In both inversions the earth was modeled with a 7 × 7 × 7 set of prismatic cells. Each elevation plane had 49 data points whose horizontal locations coincided with the horizontal midpoints of the model cells. In the inversion of the data at the lowest level, there were 49 data points and 343 cells, and so the inverse problem was underconstrained. Fedi and Rapolla constructed a model by solving the linear system using SVD. No regularization or positivity was incorporated, and the recovered magnetic susceptibility was concentrated near the surface. This is in accord with the results of Li and Oldenburg (1996) where they show that a surface concentration is a natural consequence of finding a susceptibility model that reproduces the data and minimizes $||\kappa||^2$. The SVD solution intrinsically generates a solution with a minimum l_2-norm and, hence, the comparison is completely valid.

In contrast to the above, for the inversion of 10 levels of data (for a total of 490 data points), there were more data than unknowns, and hence it was possible for Fedi and Rapolla to (nearly) recover exact values for the 343 parameters. But this merely shows that data at other levels can provide independent information compared to the 49 data points at the lowest level. In fact, the 49 data points at the lowest level form an incomplete set because of the coarse sampling and also because of their limited areal extent. The same complementary information can be supplied by obtaining other data on the original data plane. In Figure 1a, we show the magnetic anomaly from Fedi and Rapolla's example and 480 locations at which the anomaly has been sampled. An SVD solution, exactly paralleling their computations, is shown in Figure 1b. The anomaly appears at depth, just as they obtained by inverting the multilevel data. It occurs however, because the model space has been so restricted that, rather than solving an inverse problem, the actual computation is to estimate values of M parameters from N data equations under the condition of accurate data and $N > M$.

The second example in Fedi and Rapolla is somewhat different because the number of cells used to represent the model is now larger than the number of data points. We begin by re-examining their line of reasoning. In the authors' 1D example (Fedi and Rapolla, 1995), the data at different levels are available in the smallest possible area: only a single point measurement is available. Therefore, any datum at a different level provides extra information. Thus, the extra information is provided by truly independent observations, and not fundamentally because the observations are at different heights. It should be noted that a set of single observations at different heights contains the same amount of information as does a set of data points located along a horizontal line just above the ground. Furthermore, the reason a 1D inversion with high resolution can be performed is that the authors have restricted the model to be a column of cells beneath the observer. Given that the data are also varying independently with one spatial coordinate, there is enough information in this situation to resolve the variation of the model as a function of depth. However, this cannot be extended to general 3D cases where the data map varies with two independent spatial coordinates, but where the source distribution is 3D. In such cases, there is not enough information to determine an arbitrary source distribution. The fact that there are infinitely many equivalent sources, at different depths below the data map, is the best illustration of the point.

We belive the authors erred when trying to extend the 1D result to general 3D cases. The example used to illustrate this extension is shown in Figure 7 of their paper. However, the overriding factor in their example is the explicit restriction that the model has a known strike length and that the physical property does not change along the strike. Once these restrictions are applied, the model varies only with two spatial coordinates. Consequently, the inversion is strictly a 2D problem, not a 3D one, and one data map at any level contains sufficient information to reconstruct the model. Multilevel data are not needed to achieve what the authors did.

As an illustration of this point, we emulate the authors' example and demonstrate that a good inversion can be performed to provide depth resolution with one map of data. We invert a subset of the data shown in Figure 1. Only the 196 data points in the central portion of that data map which are directly above the model in Figure 1b are used. The inversions are carried out with a 3D algorithm that again minimizes the l_2-norm of the susceptibility (Li and Oldenburg, 1996). No additional information such as positivity or depth weighting is incorporated. We use this algorithm to perform two inversions. We first invert the data to construct a 2D model with the known strike length (which is termed 2.5D by Fedi and Rapolla). The inversion uses 700 square cells of 50 m on a side, so we are working with an underdetermined problem. The recovered model is shown in Figure 2a and the anomalous block is well imaged. We next invert the data to construct a 3D model without any restrictions on the model geometry. Figure 2b is one cross-section through the recovered 3D model, which is composed of 1960 cells with dimensions 125 m × 100 m × 300 m; it does not show any resemblance to the true model. Although results are not reproduced here for brevity, numerical experiments have shown that including additional levels of data in the unconstrained 3D inversion does not improve the result either. Therefore, we submit that the good result obtained by Fedi and Rapolla (1999) is not from the use of multilevel data, but from the restriction that is placed on the model.

In summary, the upward-continued data at different heights do not improve the depth resolution of the gravity or magnetic inversion. Any resolution improvement can only come from the use of prior information about the distribution of the density contrast or the magnetization. One form of prior information is the restriction on the geometry of the distribution. What the paper by Fedi and Rapolla has illustrated is the effect of adding prior information, not the effect of adding upward-continued data.

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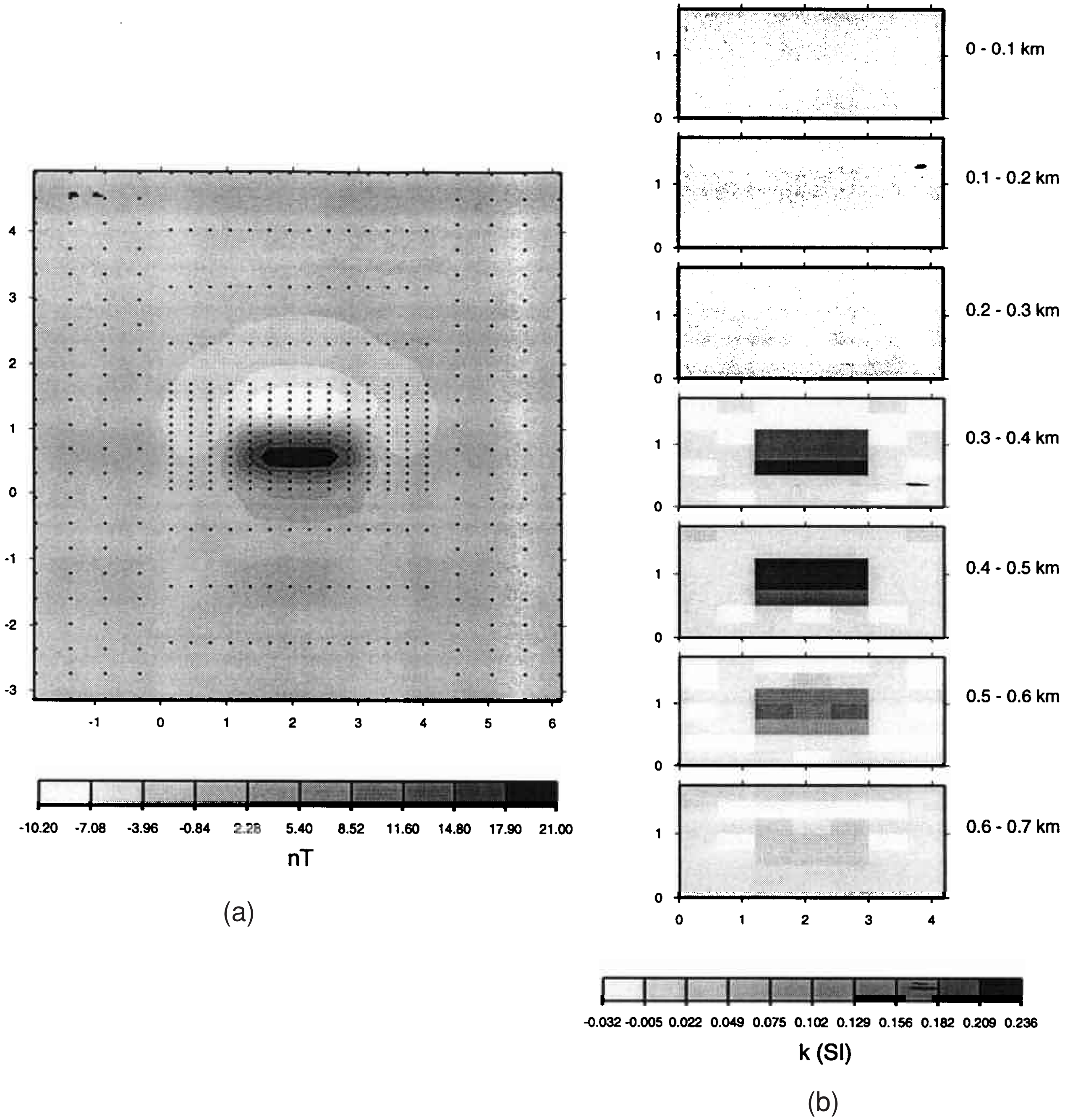

FIG. 1. (a) The magnetic anomaly from a buried prism when the inducing field has declination 0° and inclination 56°. The prism is 1800 m × 750 m × 300 m and is buried at a depth of 300 m. The dots indicate locations at which accurate field data are measured. The sampling strategy is designed to capture roughly the same information that would be contained in the 10 data levels. This necessitates sampling more frequently in the original 7 × 7 data grid and also acquiring data outside the initial grid area. (b) The susceptibility for the seven planes of the model domain. Compare this figure with Figure 4a of Fedi and Rapolla.

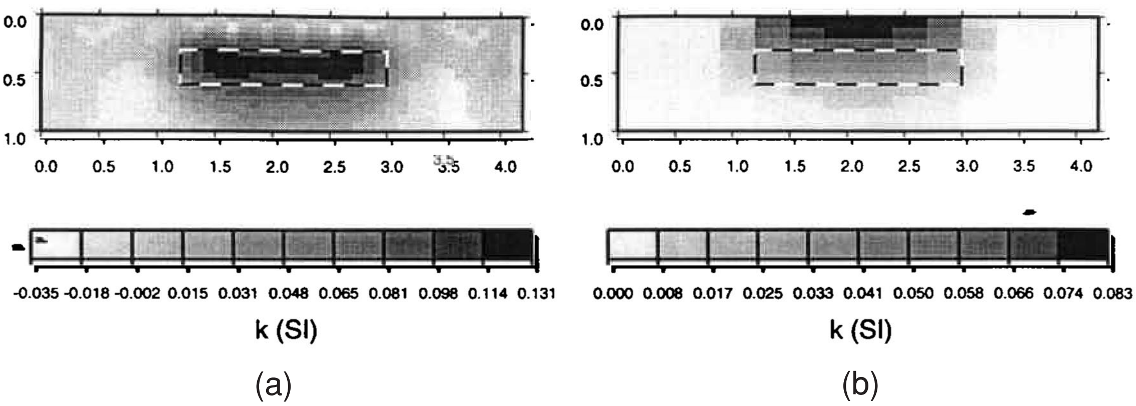

FIG. 2. Results of inverting the data in Figure 1 using different assumptions. (a) The recovered 2D susceptibility obtained by using the correct strike length. (b) One cross-section through the center of the recovered 3D model. The 3D inversion has allowed the susceptibility to change in all directions. The white box outlines the position of the causitive block in each section.

On "3-D inversion of gravity and magnetic data with depth resolution" (Maurizio Fedi and Antonio Rapolla, GEOPHYSICS, 64, 452-461).

Fedi and Rapolla (1999) state that potential-field inverse problems can be improved by using data collected at different levels above the source. This result seem to contradict Gauss' theorem, which states that a harmonic field (e.g., the gravitational potential) is uniquely determined by its values on a surface surrounding the sources. Although we do not disagree that discreetly sampled data in different levels contain some additional information as compared to data collected at a single level, we believe that such additional information cannot qualitatively improve the fundamental problem of potential-field data inversion, i.e., its inherent ambiguity.

For a formal analysis of the ambiguity problem we refer to Strykowski (1997) and Boschetti et al. (1999). Here, we emphasize, using simple examples, the crucial importance in potential field inverse problems of (1) the algorithm employed in the inversion process, (2) the horizontal extent of the data collection, and (3) the problem parameterization. We believe that these factors, not the presence of data at multiple levels, are responsible for the high quality of the results presented in Fedi and Rapolla (1999).

Inversion algorithm. In Figure 1, we reproduce the results showed by Fedi and Rapolla (1999), except for the gravity problem. All the numerical experiments presented here have been performed in 3D, following the same parameterization as in Fedi and Rapolla. However, we show the results as a 2D vertical cross-section in the *y*-direction through the center of the area. Fedi and Rapolla tackle the (underdetermined) inverse problem using Penrose's pseudoinverse solver. As with other algorithms used for geophysi-

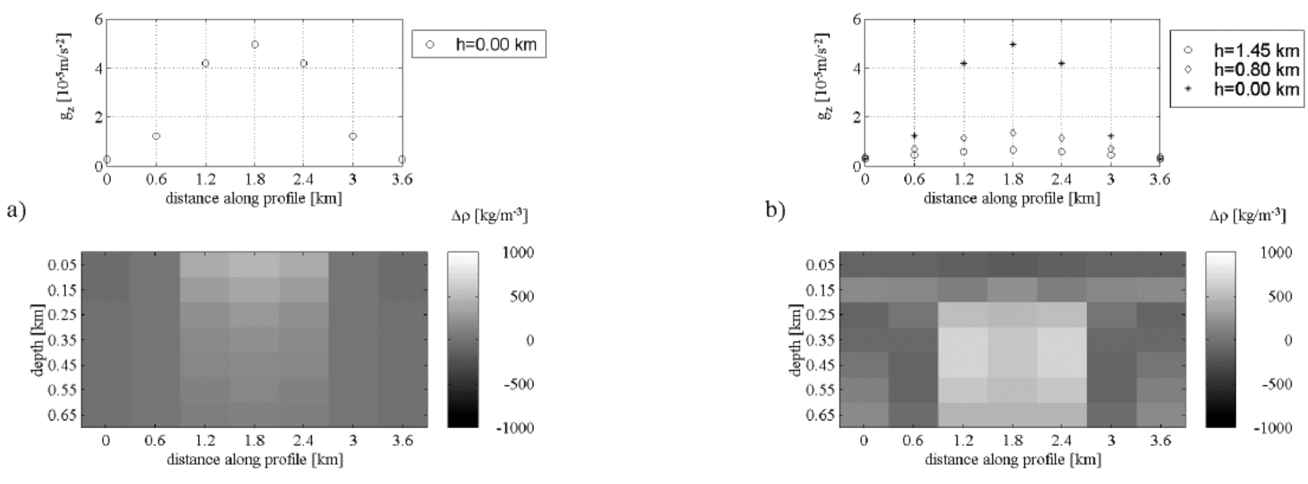

FIG. 1. Inversion of data collected at a single level (a) and at three different levels (b) as in Fedi and Rapolla (1999). (We applied a mass density contrast of the initial body of 1000 kg/m³.)

cal inversion, such an operator solves for a minimum (l_2) norm solution. Minimum norm not only implies a certain degree of smoothness but, more importantly, implies that solutions of "smaller amplitude" will be favored. In the case of potential-field data, this means solutions characterized by smaller mass density/susceptibility contrast. Furthermore, these solutions tend to concentrate anomalous mass density/susceptibility close to the measurement level, because at this depth a weaker source (i.e. with a smaller l_2-norm) is sufficient to reproduce the data.

Consequently, the concentration of anomalous material close to the surface in the inversion of potential-field data collected on a single level (Figure 1a) is merely the result of the algorithm employed in the inversion. Unfortunately, this point is often overlooked in the literature.

Additional information from data collected at different levels. Gauss' theorem states that a potential field in free space can be obtained from the potential field on a surface surrounding the source. This concept is routinely used for upward/downward continuation. However, this theorem strictly applies only when the field is known at each point on a surface. In real cases, this can never be achieved. Consequently, data at different levels contain additional information about the source. In the following, we will briefly discuss the general principles and, subsequently, relate them to the method of Fedi and Rapolla (1999).

In Figure 2, we see the gravity response generated by horizontally distributed point sources buried at the same depth and measured at two different levels. Closer to the source, the support of the response is quite compact and goes to zero quite quickly (although, rigorously, the support is infinite at any level). At the higher level, the support becomes broader and the gravity response smoother. Consequently, a measurement at a higher level is affected by a broader distribution of material in the horizontal.

Why and how should this affect the result of the inversion? In a typical experimental configuration the horizontal extent of the data collection coincides more or less with the extent of the underground model. Consider the solution in Figure 1a. It is gravitationally equivalent (i.e., it gives the same gravity response) to the synthetic model used to generate the data at the locations where the field is sampled. However, if we extend the data sampling laterally, the fields due to the two sources differ. This is shown in Figure 3 (see white circles), where the difference between the field due to the solution in Figure 1a and to the initial model used to generate the data are shown on a data sampling extended two times (in both *x*- and *y*-direction). The data domain of Figure 1a is in the center of the extended data domain. The two fields coincide (i.e., the difference is zero) just above the calculation domain, but differ to the sides, as expected. In the same plot we see the difference between the field due to the solution in Figure 1b (i.e., the one obtained by inverting the data collected at three levels) and to the initial model (dark dots). The two fields again coincide above the calculation domain, but their difference is now very small to the sides as well.

This shows that data at higher levels give information on the lateral continuation of the data at lower levels. Roughly speaking, using data at multiple levels is equivalent to using data at a single level, but with a larger horizontal extent. This is confirmed in Figure 4a. Here we invert for the same configuration as in the Fedi and Rapolla case, but the horizontal data extent is twice as large (in both *x*- and *y*-directions). The

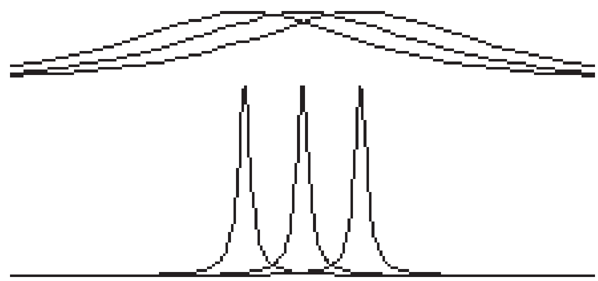

FIG. 2. Gravity response at different levels due to point sources. Measurements at higher levels respond to wider distribution of material.

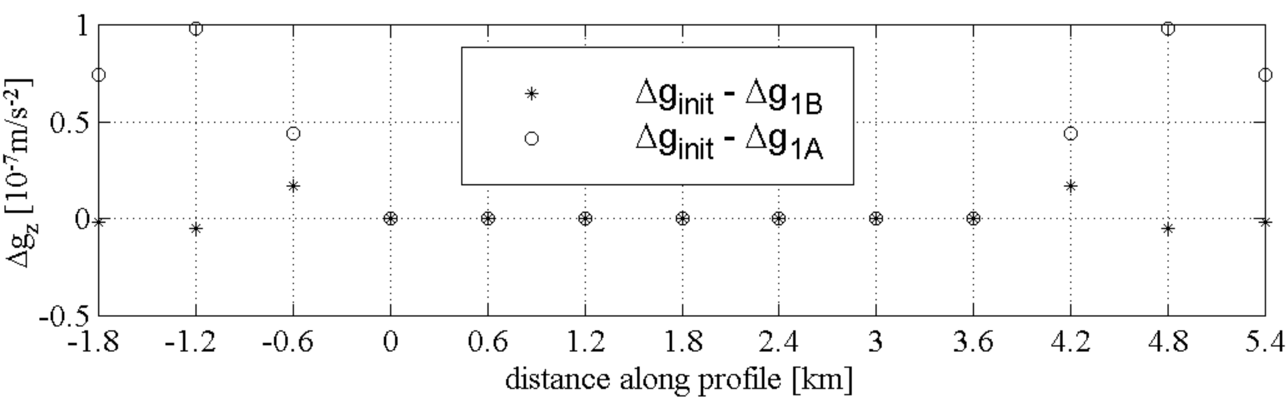

FIG. 3. Difference between the fields due to the solution in Figure 1a and the initial solution at sampling locations extended laterally compared to the calculation domain (white circles). Same for solution in Figure 1 (dark dots). Solution 1b gives a field very close to the one from the initial solution, even to the sides of the calculation domain.

source appears to be located at depth, as in the case when data at more levels are inverted (see Figure 1b).

The effect of parameterization. Another question could be asked: "Why is the minimum norm solution in Figure 1a close to the surface while in Figure 4a (i.e., when data are collected on a wider extent) the minimum norm solution is deep? What is the difference between the two configurations?" The difference lies in the extent of the calculation domain. In Figure 1a, the calculation domain has the same horizontal range as the data collection, whereas in Figure 4a, the calculation domain has a smaller horizontal range, i.e., we "squeeze" the solution and force it to be located at the center of the data extent. This forces the solution to have different values and shifts the "minimum norm solution" to deeper layers. When we enlarge the calculation domain in order to cover the horizontally extended data (see Figure 4b), then the "minimum norm" solution is again found closer to the surface as expected.

Reality and "reasonably looking" sources. The pragmatic skeptic could claim that "despite all of the above, the method works in practice and gives a reasonably looking solution." However, we should note that in real applications we cannot choose a calculation domain smaller than the data extent, unless we are extremely confident that the density/susceptibility contrast of the material outside the calculation domain is negligible. This is equivalent to stating a high degree of a priori knowledge about the source distribution that can only be based on the independent a priori information.

In summary, we can state:

1) Inside the ambiguity domain, what solution is chosen is determined by the assumptions in the inverse routine. Such choice will be meaningful only if the assumptions reflect some a priori known characteristic of the initial or "true" source.
2) Collecting data at multiple levels is approximately similar to collecting data over a wider horizontal extent.
3) Choosing a smaller horizontal extent for the calculation domain than the data sampling domain can only be justified by the existence of reliable and independent a priori information about the source distribution.

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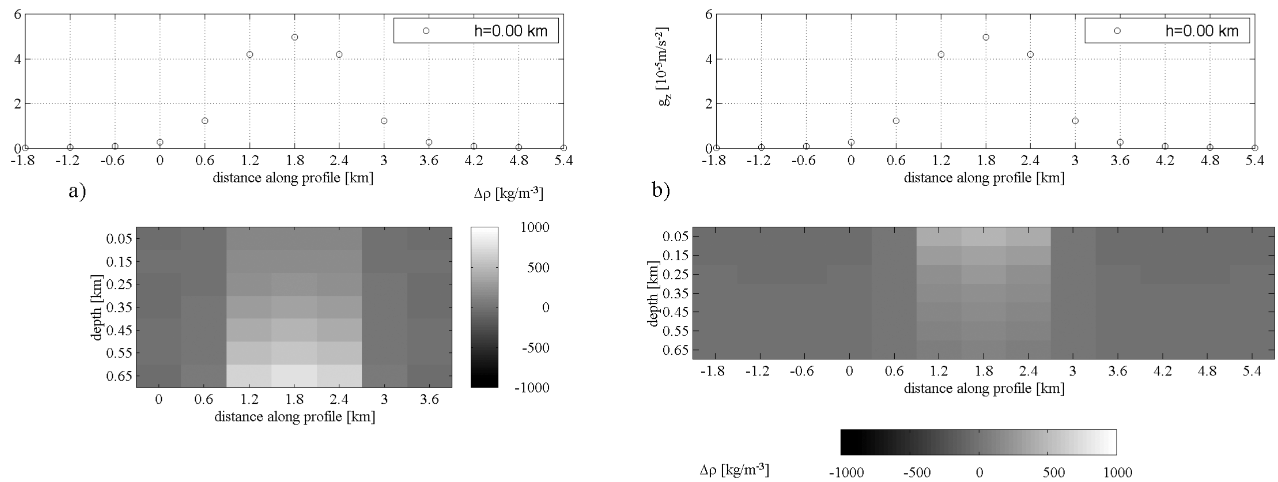

FIG. 4. (a) Solution after the data sampling is extended laterally to the sides of the calculation domain. (b) Same data sampling but now the calculation domain is also extended laterally, and the minimum norm solution is again closer to the surface.

Reply by the authors to the discussions by D. W. Oldenburg and Y. Li, and by G. Strykowski and F. Boschetti

We will be deliberately short in this reply. In fact, since the interest obtained by our work on potential-field depth resolution, we are writing new papers to give further insights into the theory.

Let us begin with the discussion by Oldenburg and Li.

We agree that our points of view are still distant, but it is surprising for us to realize that the crucial example that they use to manifest their opinions is completely wrong. To be clear, we refer to their main criticism above the multilevel data inversion, "We believe the authors erred when trying to extend the 1D result to general 3D cases." Oldenburg and Li substantiate this opinion by showing a section of an inverted 3D model (their Figure 2 at right, the source space is subdivided into 1960 cells, each size 125 m by 100 m by 300 m), which, "does not show any resemblance to the true model." To this end, they further argue that, "Although results are not reproduced here for brevity, numerical experiments have shown that including additional levels of data in the unconstrained 3D inversion does not improve the result either."

We do not know the number and the type of such numerical experiments, but our numerical experiment (Figure 1), reproducible by anyone using well-known algorithms such as SVD or CGLS, shows just the contrary for exactly the same source and number of cells assumed by Oldenburg and Li. In other words, we may also in this case confirm that within the a priori framework of a block source model, the inversion of a multilevel set of potential-fields data is likely to provide a depth-resolved solution.

We particularly disagree on their singular way of defining the restrictions imposed on a model. When referring to our examples (Fedi and Rapolla, 1999, Figure 4) they say of restriction on the model geometry, when making their test (their above discussed Figure 2, right) they prefer to speak of "model without any restriction on the model geometry." We think it rather subjective to assess the level of model restriction from the cell number, apart for limiting cases (very small or very large cells), which are far from both the above. In any case, Figure 1 definitely shows that the multilevel inversion had success as well in the case that they judge as "without any restriction." Their main conclusion is therefore based on a wrong and not objective example (their Figure 2, right).

Oldenburg and Li are critical of our example on 3D inversion (Fedi and Rapolla, 1999, Figure 4) also because the number of data equations (N) is greater than the number of unknowns (M). In this case, they think that "there were more data than unknowns and hence it was possible for Fedi and Rapolla to (nearly) recover exact values for the 343 parameters." We will now show that also in this case Oldenburg and Li do not well support their concerns. In fact, it is likely that good results also occur for $N \leq M$. Again, we take into consideration the model that Oldenburg and Li describe as "without any restriction on the model geometry." While in Figure 1, a model obtained for $N = 2548$ and $M = 1960$ was described, Figure 2 shows instead models relative to $N = M = 1960$ (Figure 2a) and to $N = 1568$, $M = 1960$ (Figure 2b). It is clear that the depth resolution is fairly good in all three cases.

We sincerely admire many papers by Oldenburg and Li, but this time we disagree with them on several other points of their discussion. First, we recall to Oldenburg and Li that we have never written *tout court* that "upward-continued data contain extra information," but that instead it happens when a priori information consists of a block source model. See the abstract, for instance, where we wrote, "The a priori information consists of delimiting a source region and subdividing it in a set of blocks. In this case, the information related to a set of field data along the vertical direction is not generally redundant and is decisive in giving a depth resolution to the gravity and magnetic methods." Therefore, after having addressed a discussion about equivalent layers and the general interpretation ambiguity of potential fields (see the introduction, page 452, around the unnumbered equation), we built our paper entirely around the above kind of a priori information, which is the most commonly used in scientific literature.

In some other points we find their arguments rather sophistic. For instance, while describing our 1D multilevel example (Fedi and Rapolla, 1995), they first observe that, "any datum at a different level provides extra information," but immediately later they say that "the extra information is provided by truly independent observations and not fundamentally because the observations are at different heights."

Finally, they give an interesting example (their Figure 1) of how the depth resolution may be gained using a single level of data, and we agree. But referring to our 2D multilevel case (Fedi and Rapolla, 1999, Figure 7), which better provides a depth resolved source, they rather surprisingly state that "Multilevel data are not needed to achieve what the authors did." For us, this is just an eccentric way to admit that we did modeling with depth resolution by multilevel data. On the other hand, it is evident that their single-level example does not at all contradict our view of the problem. Instead, it is a confirmation that potential-field data, within the a priori information context of a block-source model or even other a priori information, are likely to yield a depth resolution.

Also, we carefully read the discussion by Strykowski and Boschetti. The most important aspect is surely that their results and figures strongly support multilevel inversion. However, we believe that their discussion is more focussed on single-level inversion properties and limitations instead of on multilevel ones. To be clear, they claim that the good quality of our (or of any?) inversion is related to the following factors: (1) the algorithm employed in the inversion process, (2) the horizontal extent of the data collection, and (3) the problem parameterization. We obviously agree with the importance of these issues, but would remark that, differently from what is asserted by the authors, the above factors are well known by most geophysicists and, as such, have been sufficiently described or taken into account in ours and many other papers.

But what is the actual contribution of the authors' discussion about multilevel inversion? Any reader may observe that their figures clearly show the better performance of multilevel inversion. Also, their Figure 3 is a nice example showing the superiority of multilevel inversion in predicting the data out of the domain of measurements used in the inversion, as correctly observed by Strykowski and Boschetti. But, after these points, the authors completely forget the multilevel case and address their discussion to the single-level one. In fact, what is shown in their Figure 4, is merely that single-level inversion performs better if the source horizontal extent is more restricted with respect to the data extent. Nothing more is found about or against multilevel inversion.

We have no difficulty agreeing to this single-level inversion point (also touched on by Oldenburg and Li) and to the fact that such a priori information needs to be well justified. But, as already written above, we stress that showing such properties of the single-level case does not contradict what is affirmed in our paper. On the other hand, their results (see Figure 3) are the best demonstration of the usefulness of multilevel inversion.

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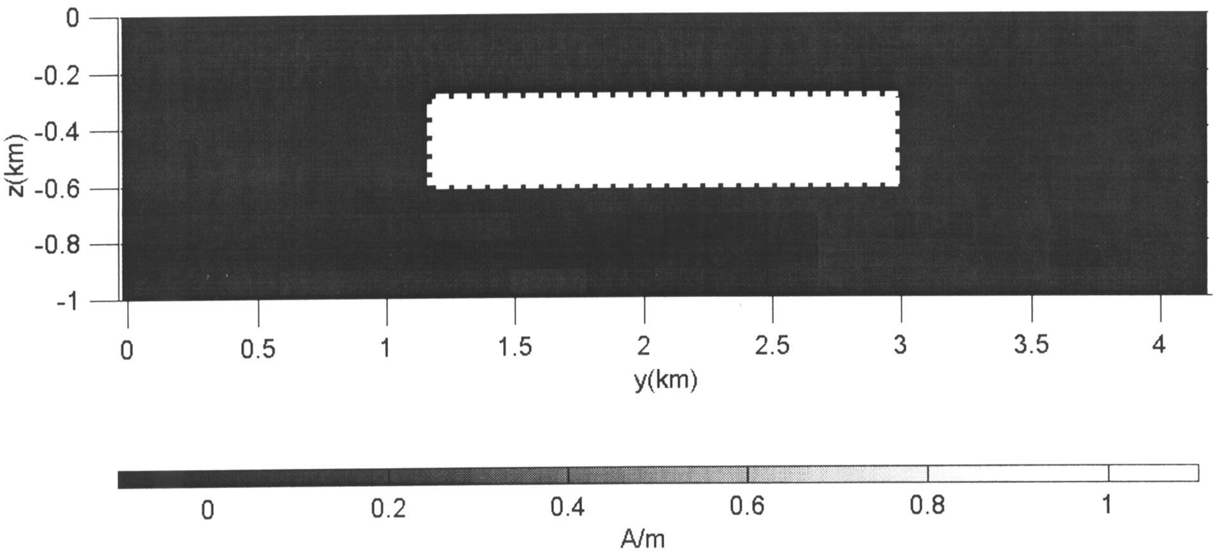

FIG. 1. The central vertical section of the same 3D body as in Figure 2 (right panel) of the discussion by Oldenburg and Li, (this issue) after a multilevel inversion. The multilevel field consists of 196 data/level for 13 levels from 0 to 1 km height, with a spacing of approximately 60 m. The source volume (and the a priori information) consists of 1960 cells, each 125 m by 100 m by 300 m. Differently from what affirmed in the discussion by Oldenburg and Li, the multilevel dataset clearly allows a depth resolution to be obtained for their "model without any restriction on the model geometry."

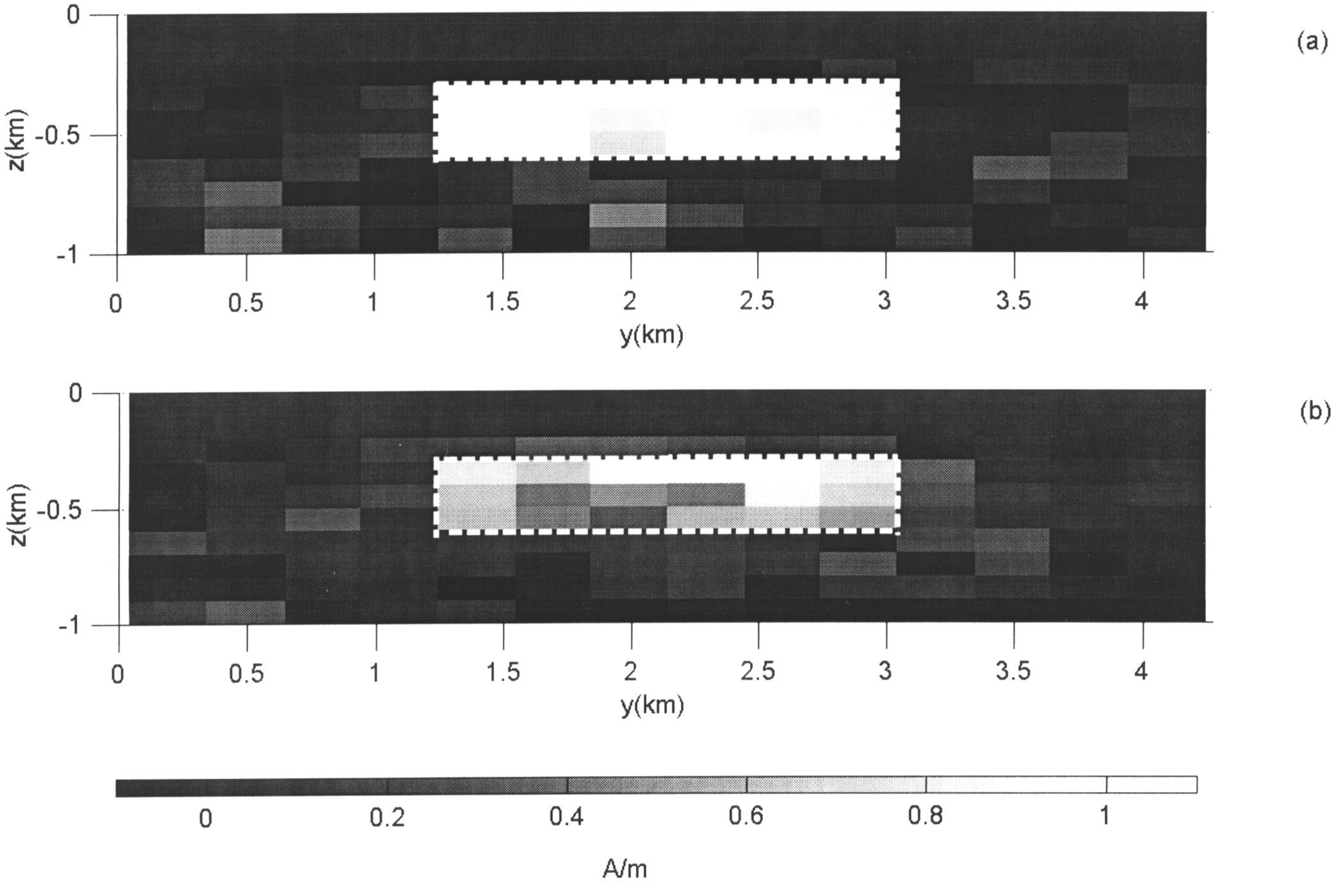

FIG. 2. (a) The central vertical section of the same 3D body as in Figure 1 but after inversion of multilevel data sets consisting of 196 data points/level for ten levels, from 0 to 0.8 km height. (b) 196 data points/level for eight levels from 0 to 0.8 km height. Differently from what is supposed in the discussion by Oldenburg and Li (this issue), the depth resolution is also obtained when the data number is not greater than the number of unknowns.