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A fractal-based algorithm for detecting first arrivals on seismic traces

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ABSTRACT

A new algorithm is proposed for the automatic picking of seismic first arrivals that detects the presence of a signal by analyzing the variation in fractal dimension along the trace. The "divider-method" is found to be the most suitable method for calculating the fractal dimension. A change in dimension is found to occur close to the transition from noise to signal plus noise, that is the first arrival. The nature of this change varies from trace to trace, but a detectable change is always found to occur. The algorithm has been tested on real data sets with varying S/N ratios and the results compared to those obtained using previously published algorithms. With an appropriate tuning of its parameters, the fractal-based algorithm proved more accurate than all these other algorithms, especially in the presence of significant noise. The fractal method proved able to tolerate noise up to 80% of the average signal amplitude. However, the fractal-based algorithm is considerably slower than the other methods and hence is intended for use only on data sets with low S/N ratios.

INTRODUCTION

The accurate determination of the traveltime of seismic energy from source to receiver is of fundamental importance in seismic surveying. This is particularly the case with seismic refraction and tomographic surveys where traveltimes, usually of first arrivals, are used to determine the seismic-velocity structure of the subsurface. To improve efficiency and speed of interpretation of such data it is common to use an automated technique for detecting seismic events, and several such algorithms have been published. As larger and larger data sets are now being used for such interpretations, these automatic methods of detecting seismic arrivals have become an essential part of the processing of seismic data.

Fundamentally, detection of first-arriving seismic data reduces to determining the time when the seismic trace ceases to be composed entirely of noise and also starts to include seismic signal. When such an operation is carried out manually, a subjective decision is made based on the change in the nature of the trace in terms of amplitude and/or frequency and/or phase both within the trace itself and also relative to its neighbors. However, what is a relatively simple operation for the human eye and brain is much more difficult to define mathematically and translate into an algorithm.

Several methods for locating a first break have been published (Coppens, 1985, Ervin et al., 1983, Gelchinsky and Shtivelman, 1983, Peraldi and Clement, 1972, Ramananantoandro and Bernitsas, 1987). Most of the methods are based on identifying a particular property of that part of the trace where the first arrival occurs. Some methods also rely on comparison of the trace with its immediate neighbors. The different methods proposed to detect first arrivals will give slightly different arrival times depending on exactly what property of the trace they are based on, but, in general, are extremely effective provided there is an adequate signal-to-noise (S/N) ratio. However, in a situation of very low S/N ratio, their accuracy may be affected seriously.

In this paper, we propose a new method of picking seismic first arrivals in noisy data sets based on the change in fractal dimension within the trace associated with the advent of the signal. Since fractal dimension can be thought of as measuring the "roughness", i.e., the overall shape, of the trace, the algorithm automatically simulates the way the human brain identifies the first arrival.

CALCULATION OF FRACTAL DIMENSION

Since its original introduction by Mandelbrot (1967) the concept of fractals and fractal dimension has found widespread applications in many fields including the earth sciences. For the definition and an extensive description of the concepts behind fractals the reader is referred to Feder (1988), Kaye (1989), Mandelbrot (1977, 1983) and Mandelbrot (1983) while their

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use in geophysics is described in Turcotte (1992) and Scholz and Mandelbrot (1989).

A number of different methods have been proposed to calculate the fractal dimension of a curve, or in this case, a seismic trace. Two methods have been employed in this study: the "structured walk technique" or "divider method" (Hayward et al., 1989, Kaye, 1989) and the "Hurst method" (Russ, 1994). The two methods represent two different classes of techniques for measuring fractal dimension. The "divider method" gives a measure of the Hausdorff dimension that is related to the geometry of the object under analysis, while the "Hurst method" is an example of stochastic techniques, and it gives a measure of the statistical relationship between the dependent and the independent variables. It should be noted that these two techniques actually measure two different phenomena and they are not expected to give the same dimension when applied to the same data set (Carr and Benzer, 1991). There has been some discussion in the literature as to the relative merits of different methods of measuring fractal dimension (Klinkenberg, 1994) and in particular to the appropriate use of the "divider method" for self-affine curves, e.g., time-series data such as seismic traces (Brown, 1987, Power and Tullis, 1991). This discussion is beyond the scope of this paper, and we will use the term "fractal dimension" for the parameter we obtain using either method. Moreover, we note that our method does not rely on the absolute value of the fractal dimension of a given part of the seismic trace, but rather on the relative variation in fractal dimension along the trace. From this point of view, we consider a seismic trace simply as a digitized curve, along which the relative variation of geometrical and statistical characteristics are analyzed independent of the absolute scaling of the X- and Y-axis.

Calculation of fractal dimension using the "divider method"

The basis of the divider method is to measure the length of the curve by approximating it with a number of straight-line segments, called "steps" (Figure 1). The calculated length of the curve is the product of the number of steps and the length of the step itself. As the step size is decreased, the straight-line segments can follow the curve more closely, smaller-scale structure becomes more significant, and the calculated length of the curve increases. If the data follow a fractal model we have

$$L(r) \propto r^{(1-D)}, \qquad (1)$$

where L in the curve length, r in the step length, and D is the fractal dimension. Plotting the logarithm of the step length versus the logarithm of the corresponding curve length, a Mandelbrot-Richardson plot is obtained (Figure 1d). The slope of a line fitted to these points is related to the degree of complexity of the curve being analyzed. This slope is related to the fractal dimension by the equation

$$D = 1 - S, \qquad (2)$$

where D is the fractal dimension, and S the slope of the line (Kennedy and Lin, 1986). The slope of the Mandelbrot-Richardson plot is equal to, or less than, 0. Thus, in the case of a curve such as the seismic trace, the fractal dimension is between 1 and 2.

Figure 2 is a typical Mandelbrot-Richardson plot obtained from analysis of a seismic trace. Note that the points do not define a single straight line segment, instead four segments (A, B, C, and D in the figure) are seen. This is because of the fact that the seismic trace is not a perfect self-similar fractal. Also, the imperfect behavior of the seismic trace is related to its representation as a series of discrete samples. The accuracy of the presentation of the trace is limited by the sampling interval and dynamic range of the digitizer. If the calculation of the length of the curve is performed with a step that is too long, the main structure of the line cannot be described which then give rise to the flat section (D) in Figure 2. When the step size is much less than the sample interval, we are not able to recognize any new structure in the curve, and again a flat section (A) results. Notice that a linear interpolation between the discrete samples is used. Details about the method implementation may be found in Clark (1986). No generally accepted rules are available in the literature for the choice of the step range to employ in the calculation of the curve length, while indications can be found in Brown (1987), Kaye (1989) and Klinkenberg (1994). Klinkenberg reports one-half the average distance between adjacent points as a suggested choice for the minimum step size, while the maximum step size should be much less than the crossover distance (see Power and Tullis, 1991). Such recommendations have been employed in this study although some experimental tuning was also necessary. In the rest of the discussion we define as "compatible" a step range that satisfies the requirements just described in relation to the part of the trace under analysis (i.e., noise or seismic signal).

Even when fractal dimension is carried out using an appropriate step size, the Mandelbrot-Richardson plot may still not result in a single linear segment. Curves that give rise to multiple straight line segments are usually referred to as "multi-fractal." This phenomenon occurs when a distribution is governed by a limited number of structures, expressing themselves at different scales as in the attempt to measure the fractal dimension of a seismic trace section (Kaye, 1989). The two linear segments in the central part of Figure 2 (B and C) result from the fact that two uncorrelated components are present in a seismic trace, i.e., the signal and the noise.

If we apply the "divider method" with a step size that is compatible with the amplitude and frequency characteristics of the noise, the resulting straight line segment on the Mandelbrot-Richardson plot defines the fractal dimension of the noise. The same is true when the step size is compatible with the amplitude and frequency of the signal, with, of course, the Mandelbrot-Richardson plot defining the fractal dimension of the signal. The relative change in fractal dimension between noise (pre-first break) and noise + signal (post-first break) and its relationship to step size is illustrated in Figure 3. In Figure 3a when a section of the trace containing only noise is analysed using a step range whose logarithm varies between 0.7-1.5, the slope of the straight line segment is -0.89. When a step size whose logarithm exceeds 1.5 is used the plot is horizontal. Figure 3b shows the Mandelbrot-Richardson plot for a section of the trace containing both noise and signal. As in Figure 3a, at step sizes whose logarithms are less than 1.5 the straight line segment reflects the noise component within the trace. However, in the presence of signal, at step sizes greater than 1.5 a second, straight line is observed. According to Mandelbrot, in the part of the Mandelbrot-Richardson plot for step sizes of between 0.7 and 1.5, the slope of the two lines should be identical in Figure 3, because when two fractal sets are unified the calculated fractal dimension should equal that of the higher dimensional component. Clearly, this is not the case with the seismic trace and we note that Russ (1994) describes practical calculations showing that in such a case, the fractal dimension assumes an intermediate value.

Calculation of fractal dimension using the "Hurst method"

In the "Hurst method" the fractal dimension is calculated by determining the range of the data within windows of different size. The maximum difference observed in a window of a given size is normalized by dividing by the standard deviation of the data. If the data follow a fractal model we have

$$\frac{R}{S} \propto F^{H}, \tag{3}$$

where R is the maximum difference observed in a window, S is the standard deviation, F is a constant and H is called the Hurst exponent. The Hurst exponent is related to the fractal dimension by the equation

$$D = 2 - H, \tag{4}$$

and it can be obtained by plotting the normalized maximum difference against the window size in log-log space (Russ, 1994).

Again, as when the "divider method" is used over a range of step sizes, a straight line on the Hurst plot is to be expected only over a limited range of window sizes.

Figure 4 shows the Hurst plot for the same seismic trace used in Figure 2. The data define a straight line only at the left-hand side of the figure, i.e., for small window sizes, while for larger windows the normalized difference becomes con-

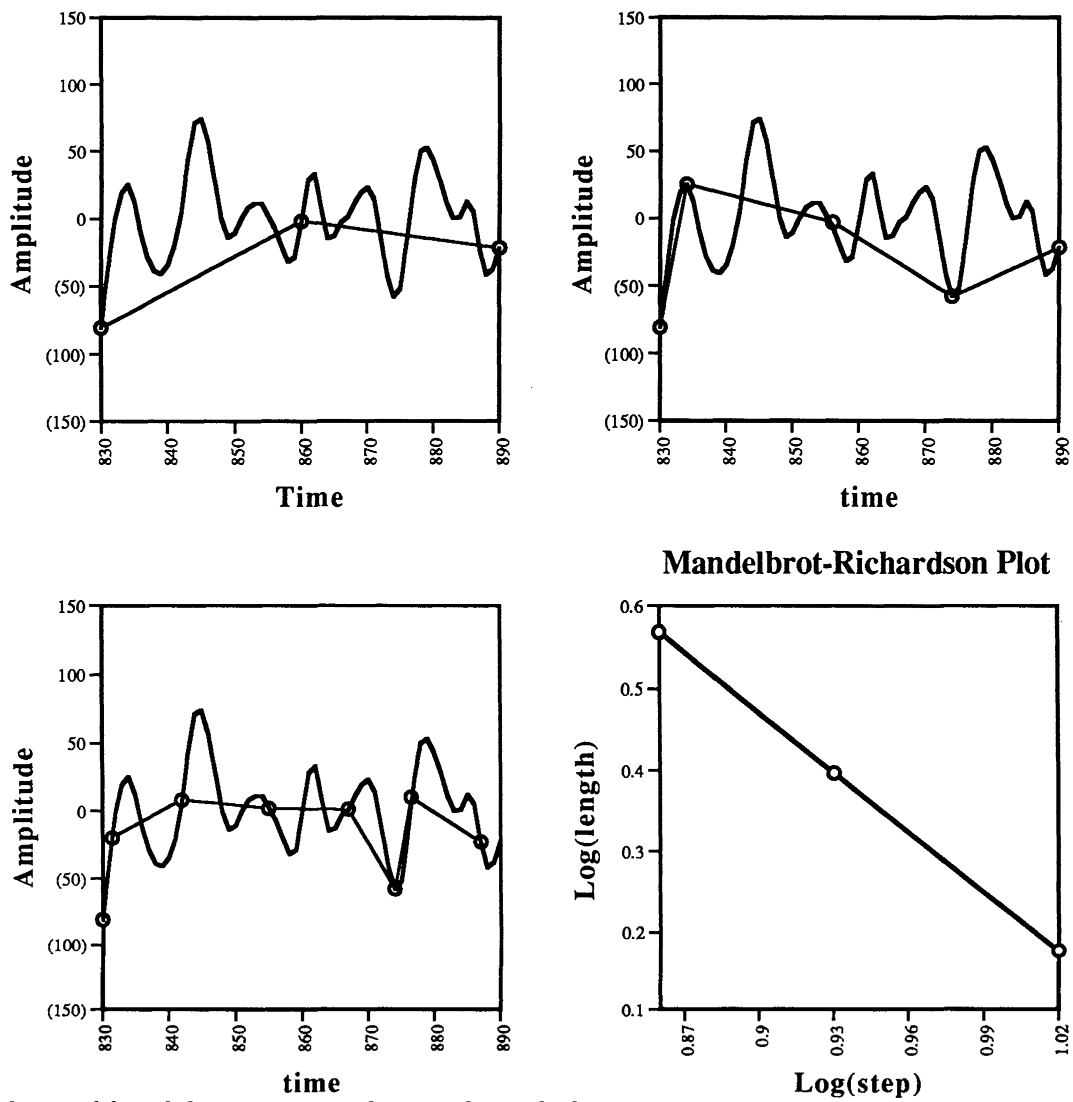

FIG. 1. Calculation of fractal dimension using the "Divider method." The curve is approximated with a number of straight-line segments, called "steps." With a long step, only the main structures of the curve are approximated, while with a shorter step, the segments can follow the line more closely. The logarithm of the step length versus the logarithm of the curve length is plotted (Mandelbrot-Richardson plot). The slope of the line fitting the points is a measure of the degree of complexity of the curve and is related to its fractal dimension.

stant. This is, again, a consequence of the seismic trace not being a perfect fractal and its representation as a series of samples. Obviously, the greatest difference that can occur within a given window is limited to the maximum and minimum amplitude within the trace. Once the points with the maximum and minimum amplitude are both contained in a window of a certain width, any larger window will not be able to find greater differences in value. Thus, all the points in the "Hurst plot" obtained for a window larger than this size will share the same value. The practical result of this observation is that for a seismic trace whose amplitude will have been resealed to lie within arbitrary limits, only a limited window size yields useful data. For instance, in Figure 4 only 9 points are significant. In some circumstances, the calculation of the fractal dimension with so few points may not be reliable.

The Hurst method has the advantage that it requires much less computation than the "divider method," and can be implemented around l-2 orders of magnitude faster. As will be shown below, it works well in high or medium signal-to-noise traces, but its performance is inferior to that of the "divider method" on noisy traces. Since the main aim of the fractal-based picking technique presented in this paper is to be robust in presence of noise, even at the cost of time, the "divider method" is preferred.

FIRST-BREAK DETECTION ALGORITHM

The basis of our first-arrival detection algorithm is that a change in fractal dimension is expected when the trace ceases to consist of just noise and begins to consist of both signal and noise.

Figure 5 illustrates how the algorithm works. First the approximate region of the trace containing the first break is selected manually. A window is then moved across this region and the fractal dimension of that part of the trace within the window is calculated. When the window is entirely before the first-arrival time, it contains only noise-window A in Figure 5. When the window includes the first break, some of the trace consists of just noise and some of signal plus noise-window B in Figure 5. When the window passes the first arrival it is completely filled by that part of the trace containing both signal and noise-window C in Figure 5. The value of the fractal dimension is calculated for each window and plotted at the location of the maximum time of the window. Figure 6 illustrates the change in the fractal dimension of the trace within the window using two different step ranges (one compatible with the noise and one compatible with the signal). The seismic trace is also shown for comparison (Figure 6a). With both ranges in step size, before the window reaches the first arrival the fractal dimension is almost constant. When the window reaches and passes the first-arrival time, the fractal dimension changes quite rapidly before again assuming a near constant value. The absolute value of the fractal dimension measured on different traces may vary, depending on the S/N ratio, on the amplification of the signal and on the sampling frequency, but the overall shape of the fractal-dimension curve is the same. It is interesting that depending on the range of the step size there may be either an increase or decrease in fractal dimension associated with the presence of signal. This depends on whether the range in step sizes is compatible with the noise or the signal. However, for the purposes of detecting the first arrival the nature of the change is unimportant.

The plots in Figure 6 showing the variation in fractal dimension along the trace are characterized by three distinct

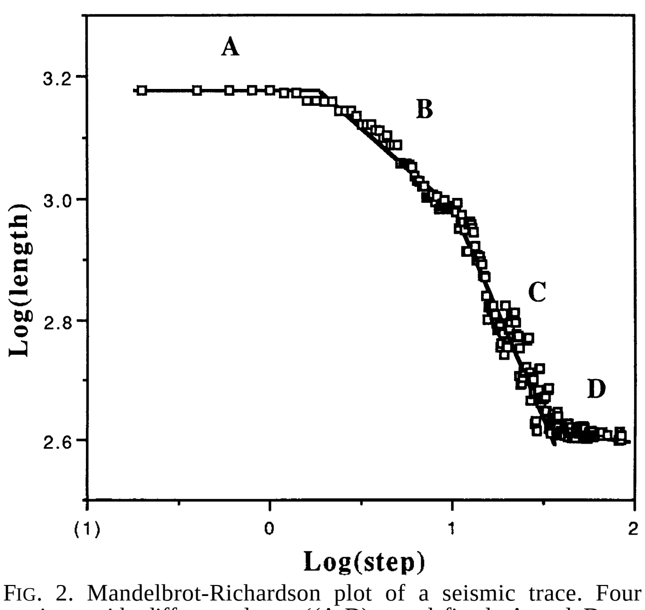

FIG. 2. Mandelbrot-Richardson plot of a seismic trace. Four sections with different slopes '(A-D) are defined. A and D are sampling artifacts while B and C are caused by signal and noise components within the trace.

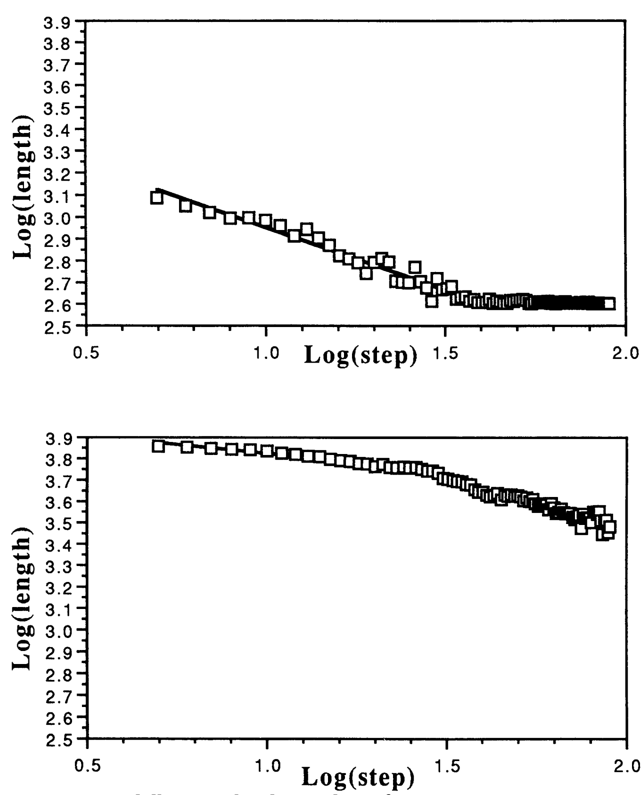

FIG. 3. Mandelbrot-Richardson plot of a seismic trace containing (a) only noise and (b) signal and noise. Where the log(step) is in the range 0.7-1.5, the "noisy" section has a higher fractal dimension. Where the log(step) is in the range 1.5-2.0 the situation is reversed.

segments: a flat segment (A) indicating the fractal dimension of the noise, an inclined segment (B) associated with the change in fractal dimension, and a second flat segment (C) associated with areas where the signal is dominating the trace. The intersection between the first flat segment (A) and the steep segment (B) occurs a few steps after the first-arrival time. This is because the algorithm needs a few points to detect the presence of the signal. The delay between the intersection of the two segments and the first-arrival time rarely exceeds a signal wavelength. This means that to detect the first arrival we can determine the intersection of these two segments (A and B), then run backwards along the trace until a local amplitude extreme is found. If required, the delay between the first amplitude extreme and the first break can be determined using traces with a high S/N noise ratio, and subtracted from the arrival time determined by the algorithm. More sophisticated methods, taking into account the correlation with adjacent traces may also be implemented.

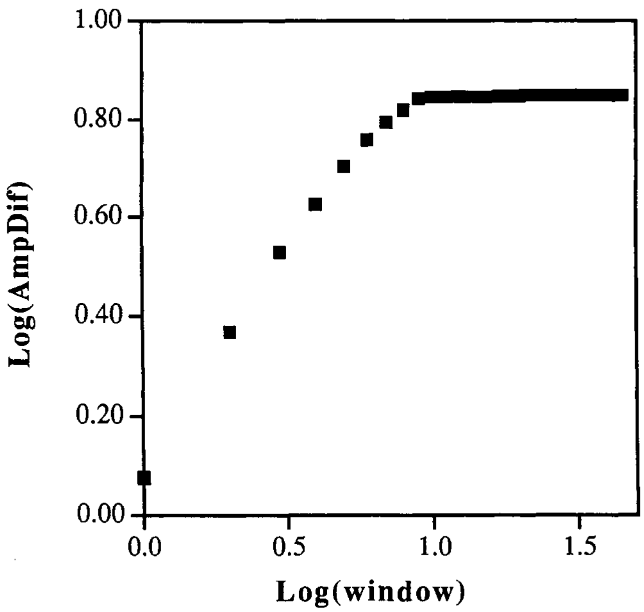

FIG. 4. Hurst plot of a seismic trace. The sloping segment at the left-hand side of the plot is caused by the fractal behaviour of the trace. The flat segment at the right-hand side of the plot is caused by the seismic trace not being a perfect fractal.

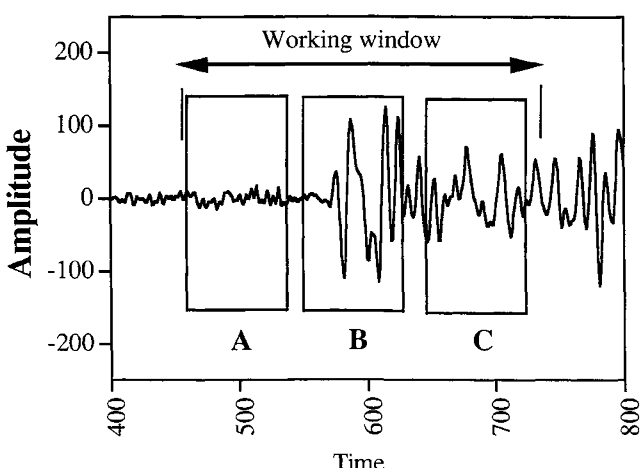

FIG. 5. Schematic illustration of how the variation in fractal dimension along the seismic trace is detected. The working window is manually selected to contain the first break. A smaller window is then moved progressively along the trace and the variation in dimension plotted as a function of the maximum time within the window.

EXPERIMENTAL RESULTS

The effectiveness of the "divider method" and the "Hurst method" based algorithms were compared with each other and with other algorithms designed to detect first breaks described in the literature. To assess their relative merits in the presence of noise, three different field data sets were used:

1) a data set, with a very high S/N ratio,
2) a data set with a medium S/N ratio,
3) a data set with a very low S/N ratio.

The first data set was collected during a seismic reflection survey across a granitoid-greenstone terrain in Western Australia [Nevoria seismic experiment, see Dentith et al., (1992)]. The second data set comes from the WISE experiment (Western Isles Seismic Experiment), an offshore seismic refraction

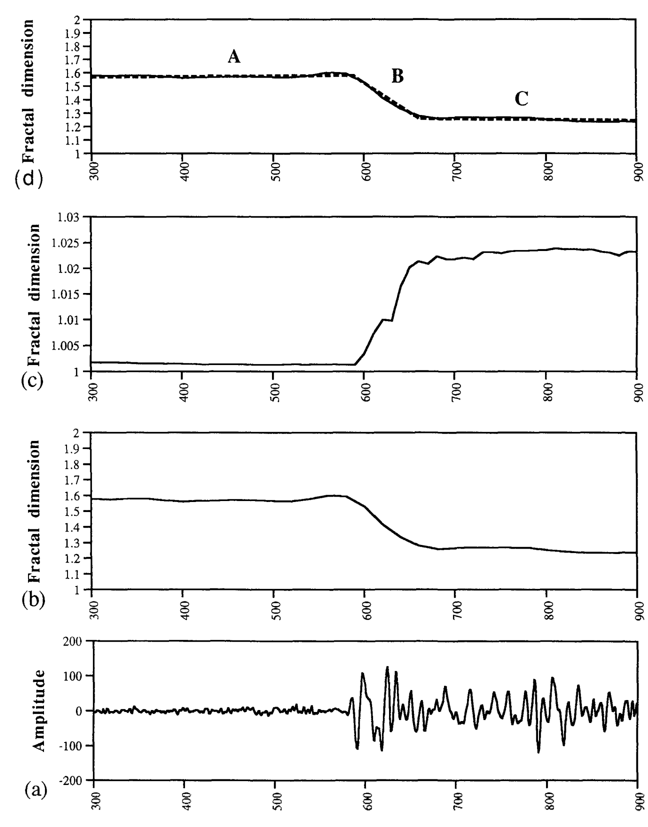

FIG. 6. (a) Seismic trace. (b) Fractal dimension of the sections to the left of a cursor moving along a seismic trace, when the investigation is carried on in a range compatible with the noise amplitude and frequency. (c) Same as in (b), but now the investigation is carried on in a range compatible with the signal amplitude and frequency. (d) Fractal dimension as in (b) approximated by three straight-line segments.

experiment in western Scotland. The last data set is part of a crustal scale refraction experiment across the southern part of the Yilgarn Craton in Western Australia (see Bolt et al., 1958). The fractal-based algorithm was able to pick the correct first arrival on the high and medium S/N data sets using either the "divider" or the "Hurst" methods. On the low S/N noise data set, the algorithm could pick most of the traces employing the "divider method," failing only on traces where even a human operator would be unable to discriminate between noise and signal. However, the "Hurst method" proved not to be effective on this data set because of the instabilities caused by the few points used to calculate the fractal dimension. In these tests the "divider method" was used for a range of step sizes and although the nature of the change in fractal dimension varied the algorithm was still successful.

The performance of the "divider method" algorithm was compared on the medium S/N data set with those of five other published picking algorithms (Coppens, 1985, Ervin et al., 1983, Gelchinsky and Shtivelman, 1983, Peraldi and Clement, 1972, Ramananantoandro and Bernitsas, 1987). Such algorithms were developed to be applicable to field data with no particular limitations, and they are representative of different kinds of picking methods available in the literature. Coppens's method is based on the detection of a sudden increase in energy on a trace, Gelchinsky and Shtivelman's and Peraldi and Clement's methods are based on different kinds of correlation with adjacent traces, while Ervin et al. and Ramananantoandro and Bernitsas's algorithms look for the first arrivals by convoluting the seismic traces with different operators. Of these, the most effective algorithms proved to be the ones from Coppens, Gelchinsky and Shtivelman, and Peraldi and Clement, whose results, together with the ones from the "divider-method" algorithm are compared in Figure 7. Only the fractal-based algorithm is able to pick the correct first arrival on all the traces. Note that, as described above, the fractal-based method detects the first amplitude extreme after the first arrival.

Progressively larger amounts of random noise were added to the high and the medium S/N ratio data sets to estimate the maximum amount of noise the fractal-based algorithm could tolerate. The algorithm was still able to detect the correct pick after noise was added, with an average amplitude of 80% of the average amplitude of the signal. However, for the algorithm to be successful in this case, a step range compatible with the signal structure is required. This is particularly important in the presence of large amounts of noise because the relative change in fractal dimension associated with the onset of the signal will be relatively small when using step ranges compatible with the noise.

Notice that the "divider method" gives a measure of the roughness of the section of the trace under analysis, that depends on the amplitude, frequency, and phase of noise and signal all together and not on any single component alone. The algorithm then detects the change in the overall shape of the curve, simulating the way a human brain discriminates the presence of signal in the seismic trace. Such discrimination is effective also in the presence of a high level of noise (see results shown in Figure 8) and does not depend on single

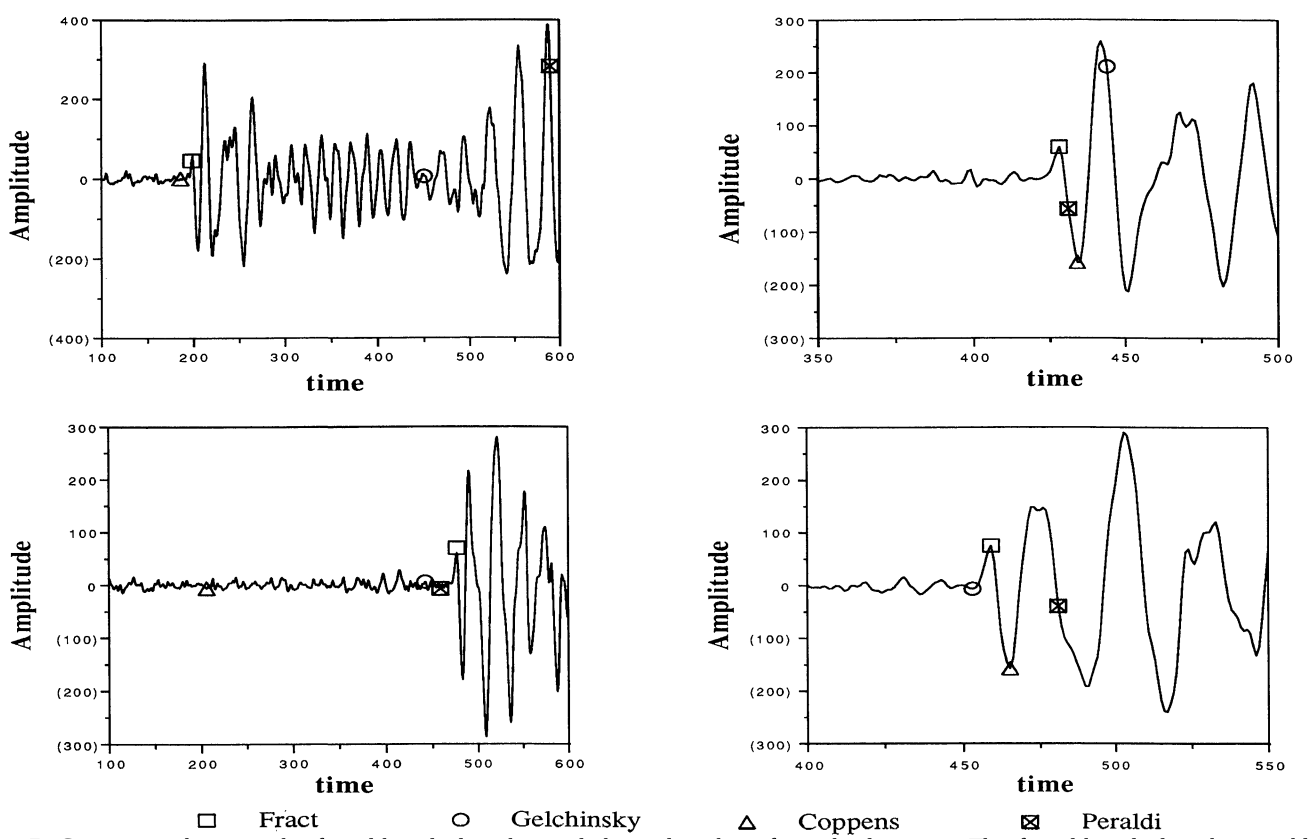

FIG. 7. Comparison between the fractal-based algorithm and three algorithms from the literature. The fractal-based algorithm is able to pick the correct arrival time in all the traces while the other algorithms may occasionally show relevant errors.

characteristics of the signal, such as frequency or amplitude. Accordingly, unlike most common picking algorithms, no preprocessing or filtering of the data is necessary.

DISCUSSION

The fractal-based picking algorithm requires a relatively large amount of calculation compared to other first-arrival detection algorithms (approximately one order of magnitude larger that the Coppens method). The fractal dimension calculation requires the measurement of the length of the seismic trace within the window for different step lengths and then regression of the points so obtained. This must be performed for a window located at each point of the trace. Also, significant effort is required to define the three segments that best fit the fractal dimension curve. The implementation of the fractal-based algorithm is quite straightforward, but both the accuracy of the result and the speed of the code depend critically on the tuning of a different number of parameters. In particular, the influence of such a tuning on the speed may be crucial, allowing performance improvements up to l-2 orders of magnitude.

Second arrivals may alter the shape of the plot in Figure 6b and 6c. In such circumstance, the flat segment corresponding to the signal fractal dimension may be substituted by a curve of a different shape depending on the characteristic of the second arrivals. However, the contact between the first and second segment in Figure 6b and 6c will be unchanged and consequently only minor modifications to the algorithm will be required to detect the first arrival. If possible, such problems could be eliminated by selecting an appropriate window and letting the algorithm run only on the section of the trace where the first arrival is known to be. In this way, valuable time would not be spent investigating useless areas.

In terms of speed, the most important parameter is the step size of the window along the trace. In the previous discussion, it was assumed that the calculation of the fractal dimension took place at each point along the trace. As described in the section, First Break Detection Algorithm, the real arrival time is determined running backward from the intersection of the two segments until an amplitude local extreme is found. Accordingly, carrying out the calculation every 5-10 points, and so doing reducing the amount of calculation of 5-10 times, does not affect the result. Obviously the maximum step allowed depends on the frequency of the signal and must not exceed the signal wavelength. An even faster calculation may be carried out with a very long window step, just to detect the area where the first arrival is located, and performing a more accurate search with a shorter step in that area. The effectiveness of such an "accelerating" process depends strongly on the signal-to-noise ratio: the simpler the trace the faster the algorithm can be run. In our experiments the parameters have been tuned on the most complex traces, and then this configuration has been used on all the traces. Another solution is to tune the parameters for fast operation using a medium com-

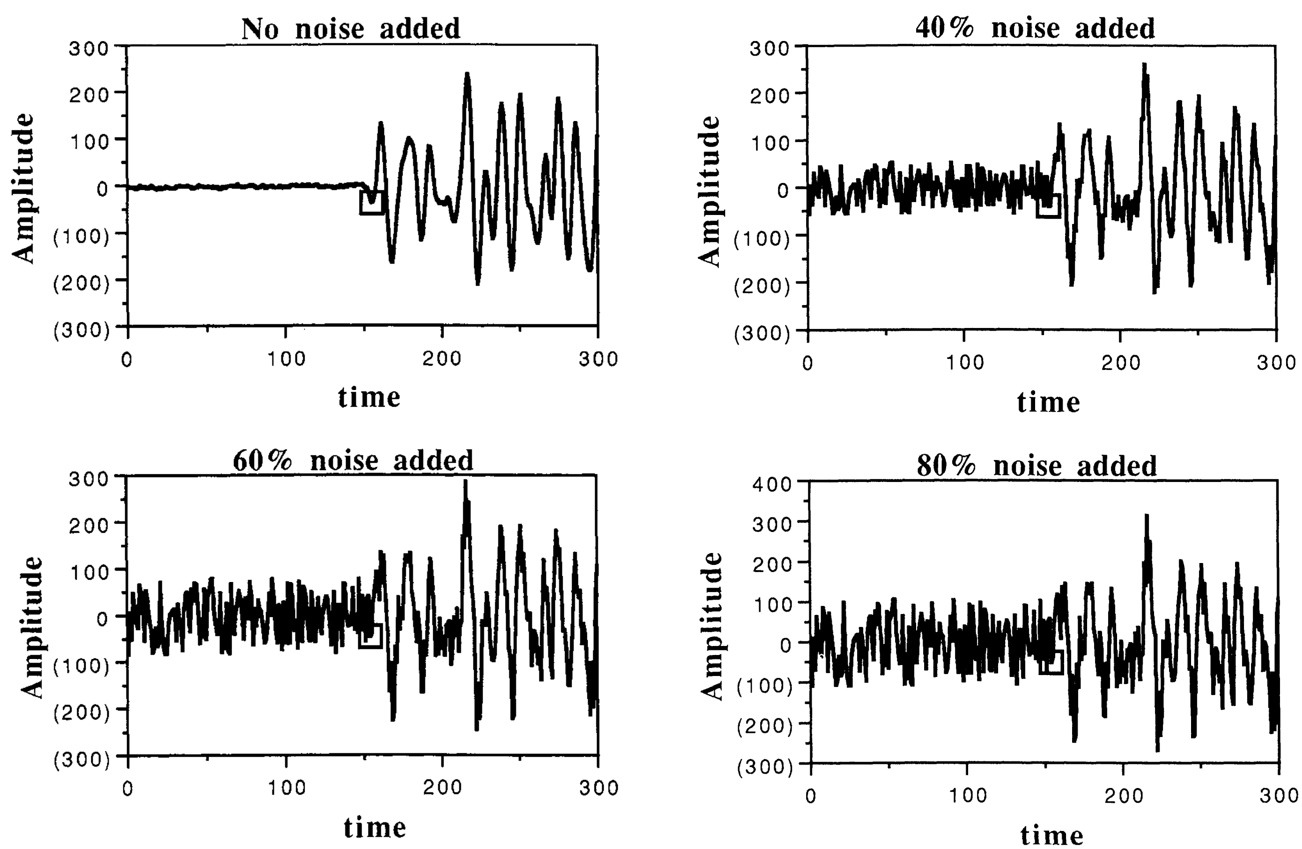

FIG. 8. Progressively increasing amount of noise added to a high signal-to-noise seismic trace to assess the maximum level of noise the code can tolerate. The fractal-based algorithm can still detect the correct pick after an amount of noise up to 80% of the average signal amplitude is added.

plexity trace and to use a slower but more accurate configuration for the hardest traces whose first arrivals were not detected.

CONCLUSIONS

The difference in fractal dimension between the part of a seismic trace containing only noise and a section containing noise plus seismic signal can be used to detect a seismic first arrival.

Analysis of the variation in fractal dimension along numerous traces highlights a consistent pattern that may be approximated by three segments. A segment associated with noise, a segment associated with the transition from noise to signal and noise, and a segment caused by signal and noise. The proposed picking method relies on the fact that the contact between the first and the second segment falls just a few steps after the first-arrival time. Different techniques may then be used to detect the correct pick-time; the most favored being running backward along the trace till a local amplitude minimum is found.

The algorithm has been tested on different real data sets and works well even when the S/N ratio is low. However, this greater reliability is achieved at the expense of speed.

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