The Geometric Modeling of Frames, Contexts and Competing Interests in a Complex Society

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Abstract

Contextual effects do not always arise from a lack of knowledge. In the complex world of socio-technical-environmental modeling, contextual effects often arise from a set of competing, and sometimes incompatible, frames. This effect is not modeled in the AI literature, but will become increasingly important as large scale agent based models of human behavior continue to be developed. This paper proposes a geometric model of such contextual frame based effects, showing that a consistent set of models can be developed for future computational implementation.

Modeling Context

Context has been extensively studied in the Artificial Intelligence (AI) Literature, with a variety of different formalizations and models suggested (Akman and Surav 1996; Brézillon 1999). While these models are undoubtedly important starting points, they tend to take an epistemic approach to context, treating it as a missing quantity of information. The assumption is that upon incorporating this extra information, a specific situation, theoretical realm, logical scenario etc. will be arrived at which can then be treated on a per case basis. Such an approach could be considered as describing situations of artificial models "knowing too little" (Brugnach et al. 2008), and there is no doubt that in many cases context can indeed be treated in this manner.

A different set of situations is not so well treated by AI; those where we might "know too differently" (Brugnach et al. 2008). Consider water management, where a variety of different stakeholders exist, each with widely competing interests. In a situation of water shortage a number of different *framings* can be provided, resulting in the attribution of different meanings to the situation, each potentially requiring different responses. A farmer will be concerned with "insufficient supply", while environmentalists might approach the water system thinking that the problem is one of "excessive consumption" (Brugnach et al. 2008). In this case, context serves as a frame for the variety of different positions, but the two contexts may be *incompatible* in that they require different actions. Farmers will clamor for more water to be released, while environmentalists will generally want environmental flows to be maintained. A modeling technology which could account for the effect of these different problem representations would benefit government and regulatory bodies that need to navigate between multiple interests and conerns.

Incompatibility has profound implications for the modeling of socio-technical-environmental systems. Probabilistic methods are ill-suited to treating incompatibility since in constructing a Kolmogorovian model we find that each context generates its own probability space (Khrennikov 2010); they cannot be compared in this framework. An axiomatic formulation is also insufficient; how are we to specify every parameter ahead of time? (Akman and Surav 1996) Indeed, this was one of the key reasons behind the drive towards using context. It is frequently the case that an important context emerges in an adaptively managed system, and this must be somehow incorporated into the model, rather than assumed a priori. The concept of *framing* is often used to incorporate context into the management of a problem (Gray 2004), with a number of equally valid, yet initially incompatible, frames specified and then policy adapted from this scenario. However, we are unaware of any agent based models (ABM) of such behavior.

In this paper we shall propose a geometrical model of contexts, showing how sets of interacting agents working within different frames (i.e. contexts) can be consistently modeled.

A Decision in Context

Humans violate many of the standard rules of probability theory (Tversky and Shafir 1992), but a number of recent works (Busemeyer, Wang, and Townsend 2006; Franco 2009; Pothos and Busemeyer 2009; Khrennikov 2010; Yukalov and Sornette 2010) have proposed that these effects can be consistently modeled in an approach inspired by quantum theory. However, to this date we are not aware of any research that explicitly deals with systems of multiple agents. In what follows, we shall construct a theoretical model that can be used to describe the behavior of a set of multiple decision makers. This model differs from more standard approaches due to a geometrical representation of the agents which makes explicit the manner in which

*Recipient of an Australian Research Council Post Doctoral Fellowship (project number DP1094974).

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shifts in context can lead to apparently spontaneous shifts of behavior. Our model differs from standard agent-based approaches, in that it gives a dynamical understanding of the manner in which a change in frame can result in a significant change in the behavior of an ABM. This implies that any model with a dynamical source underlying such a change in frame could be well modeled by this new approach.

We start with the consideration of one agent, Alice, who is attempting to decide upon a course of action. Perhaps she is trying to decide if she should extract water beyond the legal limit allocated to her. It is important to recognize that Alice makes this decision in a context, which could be changing. The most obvious context would be the amount of water available to her legally, but a number of other factors may influence her decision. For example, she could be exposed to a certain media bias, she might talk to friends, colleagues and neighbors, or she may have been impacted by the past decisions made by a sequence of governments. The current state of this context might have a very profound impact upon the course of action that our agent may choose to follow, and over a sequence of such events we might see Alice making new choices, or indeed, her original preferences becoming hard to follow, even impossible. This situation is not well represented by current modeling.

A new modeling possibility presents itself however, inspired by the q-bits of quantum theory (QT). Here, we model an undecided agent as being in a state of *superposition*, which gives a weighted sum of the two possible actions that they might choose to follow:

1. They might choose to extract a legal amount of water, we denote this option with the notation $|0\rangle$.

2. Alternatively, they might choose to extract beyond the legal limit, denoted $|1\rangle$.

Within a given context, p, the agent will have a certain probability of choosing to extract water legally. However, a change in context might change their choice, so the outcome of their decision must be considered with reference to the current context alone. QT provides a very natural way of incorporating a context into the current state of our agent:

$$|A\rangle = a_0|0_p\rangle + a_1|1_p\rangle, \text{ where } |a_0|^2 + |a_1|^2 = 1. \quad (1)$$

Here, $\{|0_p\rangle, |1_p\rangle\}$ define an orthonormal basis on a Hilbert space, the inner product of these basis vectors returns 0 or 1: $\langle 0_p|0_p\rangle = \langle 1_p|1_p\rangle = 1$ and $\langle 1_p|0_p\rangle = \langle 0_p|1_p\rangle = 0$. These basis vectors *define the current context* of our agent. Measurement of the state (1) is defined with respect to a projection operator V, where, for the two dimensional case outlined above

$$V = |0_p\rangle\langle 0_p| + |1_p\rangle\langle 1_p| = V_0 + V_1. \quad (2)$$

According to the quantum formalism, upon 'measurement', which in this case we shall consider to be forcing our agent to decide how to extract their water, the agent will choose extract legally with a probability of

$$\begin{aligned} P(l) =& \langle A|V_0|A\rangle & (3)\\ =& \langle A|0_p\rangle\langle 0_p|A\rangle & (4)\\ =& \big(a_0^*\langle 0_p|0_p\rangle + a_1^*\langle 1_p|0_p\rangle\big)\times \\ & \big(a_0\langle 0_p|0_p\rangle + a_1\langle 0_p|1_p\rangle\big) & (5)\\ =& |a_0|^2 & (6)\end{aligned}$$

and similarly, to extract illegally with a probability of $P(i) = |a_1|^2$. The assumption in (1) that the coefficients of the basis vectors sum to 1 allows for the treatment of these values as probabilities since $0 \leq P(l) \leq 1$ (and similarly for illegal extraction). This approach makes use of a geometrical notion of probability (Isham 1995). In standard probability theory, probabilistic outcomes arise from our lack of knowledge as to what has actually occurred; Alice has made her choice but we do not know what it is. In contrast, the quantum structure of probability arises through reference to the Pythagorean theorem; Alice has a current state of mind, but it may yield very different outcomes in a different context. This alternative structure can be seen with reference to figure 1(a), where the 'length' of Alice's state of mind (defined as 1 in equation (3)) is related to her probabilities of action $|a_0|^2$ and $|a_1|^2$, via a right angle triangle relationship.

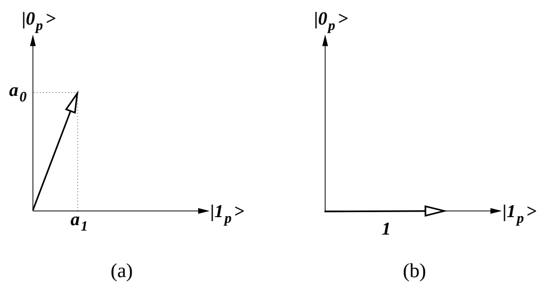

Figure 1: An agent attempts to decide upon a course of action. (a) Before making a decision they are in a superimposed state (b) after decision, they have 'collapsed' into a state of action $|1\rangle$ (or alternatively $|0\rangle$).

Probability is thus profoundly different in a quantum framework. Rather than arising from our lack of knowledge about the agent's intended action, a quantum probability might arise from a genuinely undecided agent. A context must be present and will influence our agent's choice.

We shall now make use of a key feature of the quantum formalism in the description of the agent's post-decision behavior. According to QT, the measurement of a quantum system 'collapses' the state of that system into an eigenstate of the measurement that was performed. Thus, in a quantum inspired model, if the agent chooses to extract water illegally (which has a probability of $|a_1|^2$ in context p, then after making this decision they will be in the state $|1_p\rangle$. Indeed, if we were to *immediately* ask them to decide again then we could predict with certainty (i.e. with probability 1), that they would make the same decision. This state is represented in figure 1(b).

Perhaps the most important feature of this new model arises from a consideration of context itself; it is not just a label. We can immediately develop a far richer notion of context by asking: what would happen if the context changed? QT provides us with a particularly elegant mechanism for dealing with this scenario via a change of basis. Consider figure 2, which is an elaboration of figure 1(a), and represents the changing probabilities of action that arise in the case of two different contexts, p and q. With reference to figure 2 we can quickly see that while our agent is highly likely to extract legally in context p, this has changed in context q, where they are more likely to extract illegally (since by examination of the figure we can see that while $|a_0| > |a_1|$ in context p, $|b_1| > |b_0|$ in context q).

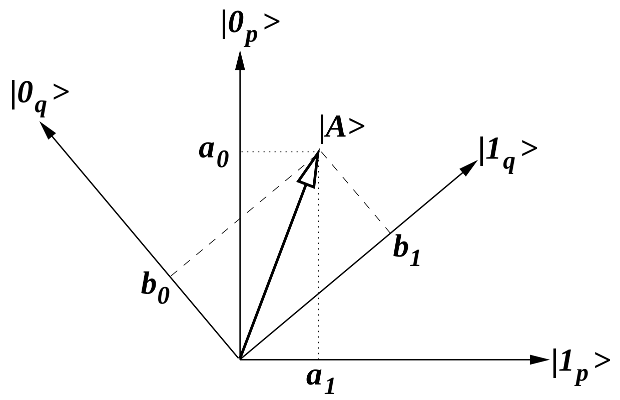

Figure 2: The changing context of a decision. The probability of choosing a particular course of action changes between contexts p and q.

What causes the context of our agent to change? They might live in a different region, be subjected to an effective media campaign, talk to new people etc. They might experience a particularly vicious drought, or it might rain. Indeed, there are a broad range of different contexts which may prove important. However, it seems possible to clearly identify two different varieties of context in this model:

Local factors pertain only to the agent in question. In the case of water management, they might include different educational backgrounds, exposure to different media sources, different preparation regimes (e.g. one agent might have previously built a dam) etc. One important subclass of local factors will result from agents interacting with one another. Thus, Alice who could have chosen to extract legally, might talk to another agent, Bob, who has made up his mind to extract illegally. In this case Alice might change her mind, thinking that: 'if Bob is extracting illegally then I should too, otherwise there will be less water available and I might miss out'. Alternatively, she might be concerned that Bob will get caught, and become more resistant to change. The implementation of such linked decisions is particularly natural in this formalism, and will be discussed at the end of this paper where we will propose that agents could be created who would tend to exhibit different behavior depending upon whether they were implementing such "follow the crowd" or "go it alone" strategies.

Global factors will result from a change to the system as a whole. It might rain, the laws might change etc. This change in context will affect all agents, but depending upon the current state of those agents it has the potential to affect each of them differently.

In what follows, we shall maintain clarity by denoting any orthonormal bases that pertain to a local context using lower case letters, while capitalized subscripts will denote global basis vectors.

Let us again return to our consideration of Alice and her decision about how much water to draw from her supply. Water management debates are frequently presented on a 'left/right' political divide which could itself be characterized as global and ortho-normal. Such a frame could be represented by using a global basis, where a left-wing ideology is depicted by $|0_G\rangle$ and a right-wing ideology by $|1_G\rangle$. Thus, this global frame might represent Alice's propensity to vote for the left-wing or the right-wing of politics in an election.

In figure 3, we see Alice making her political decision with reference to a particular instantiation of her local frame, denoted by $\{|0_a\rangle, |1_a\rangle\}$, which represents her propensity to draw water legally or illegally. This local frame can have a significant effect upon Alice's likelihood of choosing to make a particular political decision as represented in the global frame. Consider the following scenario:

1. In her local frame, Alice has a probability $|a_0|^2_{local}$ of choosing to extract legally, and a probability $|a_1|^2_{local}$ of choosing to extract illegally.
2. Let us assume that a random process, appropriately weighted by the above probabilities, determines if Alice extracts legally or not. We further assume that in this particular case the process assigns Alice to the state of extracting legally.
3. Alice's state thus 'collapses' to legal extraction in her local frame. Thus, her state is now given by $|A\rangle = |0_a\rangle$. Note that the state is 're-normalized' back to length 1 as Alice is most definitely now in this state.
4. Alice's updated state has pushed her further towards the left-wing of the political spectrum. Indeed, if every agent in the system were suddenly subjected to a vote, then we would see Alice more likely to vote on the left-wing of the political spectrum by an amount Δa_0.

A number of other effects for a single decision maker have been explored by Busemeyer and Trueblood (2010), who have used similar reasoning to model the episodic over distribution effect, which occurs in memory experiments.

Such a dependence upon decision order has many potential ramifications. If a set of agents were asked to vote in an election after undergoing a sequence of locally based decisions that were in some sense compatible with that election, then the outcome of that vote could be profoundly influenced. We note also that the phenomenon of push polling (Fox 1997) can be very easily explained in this model; a push pollster is attempting to shift the frame of their interviewees towards a desired voting outcome.

We shall now move to a more sophisticated model of two agents making their decisions with respect to both global and local contexts.

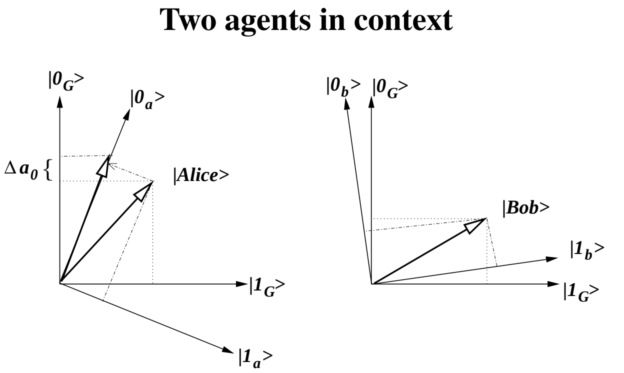

Figure 3: Two agents, Alice and Bob, must make their decisions within a particular context. This context might be global (represented by the $\{|0_G\rangle, |1_G\rangle\}$ basis), hence pertaining to all agents in the system, or local and pertaining only to the particular agent of interest. If an agent makes a decision in one context, then this might result in a shift (e.g. Δa_0) in their probability of making the same decision in another.

Explicitly adding Bob to our simple model (figure 3), we see two agents, both working within some global context G that is drawn with respect to both agents using the $\{|0_G\rangle, |1_G\rangle\}$ basis. Bob makes his local decisions with respect to his local frame $\{|0_b\rangle, |1_b\rangle\}$. This provides a very natural framework within which we can compare the different decisions likely to be made by Alice and Bob.

Each agent can be understood to be in a different state, which can evolve over time in a manner we leave unspecified for now, beyond requiring a conservation of probability. However, beyond this traditional notion of state, we have two new contexts within which to compare the decisions of our agents, and these are open to change. A key difference between this model and more traditional approaches lies in the active nature of the frames. Measurement is traditionally seen as an essentially passive process which merely polls the existing opinions and biases of a population. This is no longer the case. A measurement must be understood as a form of *interaction* between an agent's state and their current context, and this new feature allows us to model a number of new effects. In the following sections we shall illustrate three key examples: interference between decisions made in different frames; a polling steady state that can be created only with reference to a quantum process of collapse; and a non-separable interaction between agents. First however, we shall quickly discuss a new possibility that arises in the two agent model, which is not discussed in either quantum theory, or in more standard approaches to social modeling.

In figure 3 a highly non-standard possibility presents itself: the frame (local or global) in which an agent is situated can be straightforwardly understood as changing in time. A change in the frame of an agent can then be represented by a rotation of the relevant axes for that agent (i.e. $\{|0(\theta)\rangle, |1(\theta)\rangle\}$), which will change the probabilities of many outcomes. Different probabilities will arise for the 'same' decisions, which are of course no longer the same due to the change in context. When combined over systems of multiple agents, this highly intriguing modeling possibility allows for a natural consideration of the manner in which an agent's decisions will change as their context changes. Work is currently underway to investigate the agent based behavior generated by this new modeling possibility.

Interference between competing decisions

It is frequently the case that in the complex world of competing human decisions, one decision will interfere with, or even prove incompatible with, the ability of an agent to make another decision (Busemeyer, Wang, and Townsend 2006; Franco 2009; Pothos and Busemeyer 2009; Khrennikov 2010; Busemeyer and Trueblood 2010; Yukalov and Sornette 2010; Busemeyer, Pothos, and Franco 2011). For example, having chosen to extract water illegally, Bob might find himself far more likely to choose to plant rice, Alice on the other hand might find rice beyond her capabilities. Another example was presented above, where Alice became *more* likely to vote for a left-wing party after having chosen to extract legally. Such *interference* between decisions can be modeled as follows.

We recall that Alice is attempting to make two decisions: (1) whether to overdraw her water allocation, and; (2) which party to vote for in a national election. Franco (2009) has derived a general formula for the interference that two different decisions can generate for one another in the mind of their maker. For our case, this result would amount to the probability of Alice voting for a left wing party in a general election, $P(L)$, as being equivalent to:

$$P(L) = P(l)P(L|l)+P(i)P(L|i) + I(|A\rangle, V_a), \quad (7)$$

where $I(|A\rangle, V_a)$

$$= \sqrt{P(l)P(i)P(L|l)P(L|i)} \times 2\cos(\phi_0 - \phi_1) \quad (8)$$

is an interference term, that depends upon both Alice's current state and the measurement of her *local* frame, and can range from -1 to +1. This interference term accounts for violations in the formula of total probability that are frequently exhibited by humans (Busemeyer, Pothos, and Franco 2011) through its provision of an adjustment term that relates to the probabilities themselves. It has two parameters, ϕ_1, and ϕ_0 which relate to the *phase* of Alice's state. As phase intrinsically relates to the time evolution of a quantum system, we shall clarify this interference effect by adding a further elaboration to our model which allows us to discuss time dynamics.

Time dynamics and a polling steady state

We shall now add a slight complication to our model of Alice's current state, in the form of the two phase factors that were introduced in (8). These factors relate to an undetermined phase factor in (1), which could be chosen in a number of ways. One choice allows us to introduce time dependence into Alice's state of mind. Indeed, we can re-write

Alice's current state with a time dependence t as:

$$|A(t)\rangle = a_0 e^{-iE_0 t}|0_p\rangle + a_1 e^{-iE_1 t}|1_p\rangle \quad (9)$$

while still satisfying our original normalization conditions. How do these factors relate to the dynamics of the system? In fact, these new phase factors relate to the *eigenvalues* of the time evolution of the system, via the eigenvalue equations

$$H|0_a\rangle = E_0|0_a\rangle, \qquad \text{or} \qquad H|1_a\rangle = E_1|1_a\rangle \quad (10)$$

which hold as long as the dynamics of Alice's decision making is described by a time-independent operator H, which is non-degenerate. In turn, H can be used to determine the time evolution of our agent Alice, via the Schrödinger equation (Isham 1995): $\frac{d}{dt}|A(t)\rangle = -iH|A(t)\rangle$, where the interference term is now given by $I(|A\rangle, V_a) = \sqrt{P(l)P(i)P(L|l)P(L|i)} \times 2\cos\{(E_0 - E_1)t\}$. A similar set of relationships hold for the global frame, and Bob too would be similarly described. In deriving this set of relations, we have assumed that our agent can be modeled as a 'free particle' i.e. they are undergoing no interactions. This is an assumption that will need to be weakened in future analysis, which will also explore the dynamics that are possible in this picture in much more detail.

This toy model can scale to higher dimensions, which would merely involve the addition of extra eigenvectors and values throughout the analysis. Thus, this model provides a consistent and scalable way in which to model an agent trying to make a complex set of decisions which can non-trivially affect each other. In the next section we shall move onto a discussion of the manner in which those agents might affect the dynamics of *one another*. First however, we shall discuss an interesting possibility that the above time dynamics opens up within a quantum inspired modeling framework.

The quantum zeno effect (Sudarshan and Misra 1977) predicts that when quantum systems are observed too frequently they will not be able to move beyond their current eigenstate. If a state starts to continuously evolve away from a previously measured eigenstate but is quickly measured again in the same basis, then with high probability it will collapse back to its original measured eigenstate. Even if that state is not stationary, it becomes particularly stable under such constant measurement. For the current model, this effect implies that if continuously evolving agents are constantly polled, either locally or globally, then they will, with high probability, remain in their current state; they will be unable to adapt to a new situation or set of facts. If such an outcome could be found in sociological data then it would have profound ramifications for public policy. As governments attempt to adapt to a world of climate change, varying water availability, increasing fires and floods etc. it may become important to incorporate such a resistance to change into our large scale models of socio-technical-environmental systems. This idea is examined more fully in a work that is currently under preparation.

Interaction between agents

The interaction between agents is assumed to be a major local influence upon the decisions made by those agents. Let us consider Alice, as she attempts to decide at a particular point in time, whether she will extract her water legally or illegally. Alice will no doubt be influenced by the previous choices of her neighbors. We shall reflect this by considering a combined state:$|AC_{t-1}\rangle$ for all C_{t-1} in Alice's neighborhood (which we keep undefined in this paper for the purposes of generality) at the previous timestep. Thus, we must find a way in which to describe the interaction of Alice with each of her neighbors, at a given time step. However, there are a number of different choices to be made in describing this interaction. These choices are mandated by the formalism of QT itself, which suggests that superposition states can be combined in one of two fundamental manners:

Case 1: The tensor product could be used. In the case of our two agents, Alice $|A\rangle$ and Bob $|B_{t-1}\rangle$, there are a total of four possibilities obtained combinatorially through a consideration of each agent's possible actions. This list of four possibilities would be represented as follows:

$$\begin{aligned} &|A\rangle \otimes |B_{t-1}\rangle \\ &\quad = (a_0|0\rangle + a_1|1\rangle) \otimes (\alpha_0|0_{t-1}\rangle + \alpha_1|1_{t-1}\rangle) \quad (11) \\ &\quad = a_0\alpha_0|00\rangle + a_1\alpha_0|10\rangle + a_0\alpha_1|01\rangle + a_1\alpha_1|11\rangle, \end{aligned}$$

where $|a_0\alpha_0|^2 + |a_1\alpha_0|^2 + |a_0\alpha_1|^2 + |a_1\alpha_1|^2 = 1$.

Case 2: Alternatively, not every state might be available. In this case, the combined state is said to be *entangled*, and it cannot be reached through a multiplication as occurred with the tensor product. So, for example, it might be the case that Alice will always choose to do what Bob is currently doing and we would represent this using the entangled state

$$|AB_{t-1}\rangle = w|00\rangle + x|11\rangle, \text{ where } w^2 + x^2 = 1. \quad (12)$$

While the simple product state described in case 1 is relatively uncomplicated, and merely represents a combination of all possibilities, the entangled state opens up a set of rather different behaviors, which in the case of ABM can be considered as strategies. In the case of two 2-D interacting agents, six different entangled states are possible, that is, a further five strategies exist:

$$|AB\rangle = y|01\rangle + z|10\rangle \quad (13)$$
$$|AB\rangle = w|00\rangle + x|11\rangle + z|10\rangle \quad (14)$$
$$|AB\rangle = w|00\rangle + x|11\rangle + y|01\rangle \quad (15)$$
$$|AB\rangle = x|11\rangle + y|01\rangle + z|10\rangle \quad (16)$$
$$|AB\rangle = w|00\rangle + y|01\rangle + z|10\rangle \quad (17)$$

where in each case the coefficients of the relevant equation are normalized (i.e. when squared they sum to one). Every other combination yields a product state, which means that it is possible to find coefficients w, x, y, z such that the state can be rewritten in a product of form (11). This state of affairs implies that the state is *separable* or *classically compositional* (Bruza et al. 2011), whereas the above six equations imply strategies that are non-separable in a clearly defined sense. What impact would a collection of these different strategies have upon an ABM if played by a number of agents? We shall examine this point in a future more detailed work discussing the computational implementation of the model proposed in this paper.

A geometrical model of social decisions

In this paper we have presented a general model of agent based decision making where differing local and global contexts can be very naturally incorporated into the probability that a given agent will choose a particular action. Obviously, this model has been presented in a very general manner, and while we developed one particular example for the purposes of clarity, there is no reason for this new geometrical class of models to apply solely to decisions that are made with respect to water management. More detailed models and predictions will result from a consideration of particular cases and systems, but this model was presented in such a way that it could be easily extended by other researchers to their particular problem domain. We shall now conclude with some more general comments.

We start with the observation that this quantum inspired approach is not completely new to computer science, as a similar set of models are currently being proposed in linguistics (Bruza et al. 2009) and information retrieval (Van Rijsbergen 2004). A key feature of this class of models is their dependence upon a similar geometrical notion of probability to that applied here; the statistics of these models is not due to a lack of knowledge, but arises from the fundamental dependence upon context exhibited by the systems they are describing.

Due to space limitations, we have not extended the model to multiple agents, however, we note that the standard set of choices regarding agent communication remain for this model. Placing agents upon different grids, connecting them differently, and allowing them to move around, are all features capable of influencing the dynamics of this model. However, an extra set of choices are available to this set of models, provided by the different notions of interaction that are adopted for a specific case. For example, if agents interact with n neighbors according to a particular entangled strategy (12–17), then this is likely to yield different group dynamics from a set of separate agents who can make the full range of decisions depicted in (11).

In the context of scientific advice to policy making, this approach may provide an unlikely avenue of communication between natural scientists and engineers on one side and social scientists on the other. In this arena, effective communication between these groups is often challenged by a crucial difference in the approach to knowledge; while natural scientists and engineers are trained to think that there is a *truth* which needs to be discovered, social scientists tend to believe truth is a metal construct and thus contextual. In the first case uncertainty arises from not knowing the truth (knowing too little), in the second about choosing which truth to accept (knowing too differently). The latter may appear not amenable to rigorous formal analysis and is often refuted by natural scientists. The Hilbert space representation employed in figures 1–3 may allow natural scientists and engineers to model, and thus more easily accept, the view social scientists hold so dear. Similarly, the adoption of such a framework may provide confidence to social scientists that some important aspect of social theory can be considered in quantitative models, making them to relevant to the real world problems they address.

Overall, we feel that the proposed new class of models offers a promising avenue for research in AI. They allow for the sophisticated modeling of humans working within the many frames and contexts that affect their decisions, votes and choices. Such models are likely to prove essential for including the actual dynamics of human behavior in the large scale socio-technological-environmental models currently being deployed in a bid to predict the future world that we will find ourselves inhabiting.

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