Social Tagging, Guppy Effect and the Role of Interference:
A Quantum-inspired Model for Tags Combination

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Abstract. Social tagging systems are shown to evidence a well known cognitive heuristic, the guppy effect, which arises from the combination of different concepts. We present some empirical evidence of this effect, drawn from a popular social tagging Web service. The guppy effect is then described using a quantum inspired formalism that has been already successfully applied to model conjunction fallacy and probability judgement errors. Key to the formalism is the concept of interference, which is able to capture and quantify the strength of the guppy effect.

1 Introduction

Folksonomy (also known as collaborative tagging, social tagging and social classification) is the practice of collaboratively creating and managing tags to annotate and categorise content. It describes the bottom-up classification systems that emerge from social tagging. Folksonomies became popular on the Web around 2004 as part of social software applications including social bookmarking and annotating photographs. Tagging, which is characteristic of Web 2.0 services, allows non-expert users to collectively classify and find information with a few simple operations [11]. Some websites include tag clouds as a way to visualise tags.

From a theoretical point of view, folksonomies form a new area of research, where theoretical perspectives and relevant research methods are only now being defined [12]. Folksonomies are often criticised because their lack of terminological control may produce unreliable and inconsistent results. If tags are freely chosen (instead of taken from a given vocabulary), synonyms (multiple tags for the same concept), homonymy (same tag used with different meaning), and polysemy (same tag with multiple related meanings) are likely to arise, lowering the efficiency of content indexing and searching [5].

A challenging problem is the *the combination of tags*, where combined tags might refer to a different concept than the simple intersection of the original tags. For example, let us consider the categories *science* and *fiction*. Their conjunction *science fiction* represents something different from the intersection: the original concepts have been overextended. On the contrary, if we look for *science fiction* in a social tagging system, we will find simply the set of pages tagged as both science and fiction. Thus, if on one hand the folksonomies seem a very natural

and spontaneous way to produce categories, on the other hand the combination of tags, performed as intersection of the original tags, doesn't seem to be fully adequate to describe human's reasoning. Such important problem is somehow known, and there are at least two attempts to overcome it:

1. The *tagmash*, proposed by Spalding and implemented in the LibraryThing Web service[1], offers the possibility to create persisting combination of tags to form meaningful and representative clusters. Tagmash is however a semi-automated process, where the system computes the statistics, while users have to define the meaningful clusters. The list of all the tagmashes available in LibraryThing evidences the emerging of new concepts from the combinations of tags. For example, the tagmash "alcohol, history" (`http://www.librarything.com/tag/alcohol,history`) represents the simple intersection of books tagged as alcohol and history, but it evidences that users consider such combination representative of a particular set of books.
2. The *statistical clustering* emerging from the co-occurrence of tags, used for example by Flickr[2] [10]. For instance, the url `http://www.flickr.com/photos/tags/goldfish/clusters/` presents clusters of images related to the tag *goldfish*. One of those clusters contains the tags *fish* and *pet*, which we will show to produce interesting combination effects.

In this paper we make two main contributions: the first is to connect the combination of tags to an important cognitive heuristic for the combination of concepts, the guppy effect [6, 7]. In section 2 we formalise such intuition by defining the *tagged-guppy-effect* and we show experimental evidences of its presence. The second contribution is the use of a quantum-inspired formalism, which has been proposed to model the conjunction fallacy (a cognitive anomaly strictly connected with the guppy effect). Specifically, in section 3 we show how this formalism is able to capture and quantify, in a simple and intuitive way, the strength of such effect [2, 3, 9]. Finally, the paper concludes in section 5, where we summarise our work and suggest directions of future research.

We conclude this introductive part with an important note: our attempt to use a quantum-inspired formalism does not entail that we are claiming the brain to be a quantum computer; rather we only use quantum principles to derive cognitive models, leaving the mathematical principles of quantum probability detached from the physical meaning associated with quantum mechanics. In fact, the quantum formalism that we propose introduces novel mathematical tools and concepts, more suitable to describe the tagging systems and the concept of contextuality.

2 The tagged-guppy effect

Since the advent of prototype theory [8], it is well known that different members of the same category may have different degree of membership relevant to that

[1] `http://www.librarything.com/`
[2] `http://www.flickr.com/`

category. The fuzzy formalisation of prototype theory [14] assumes that the fuzzy set theory provides a framework to formalise such graded membership so to expand classical set theory[3] by allowing membership in a set to take any real value between 0 and 1. The more typical an entity is for a category, the closer its membership value is to 1; the less an entity is related to the category, the closer its membership is to 0.

However, Osherson and Smith [7] showed that prototype theory, when formalised in terms of fuzzy-set theory, contradicts strong intuitions people have about conjunctive concepts. Following this argument, they concluded that prototype theory, at least when formalised in terms of fuzzy-set theory, cannot account for graded membership in conjunctive concepts. For example, the word *guppy* results to be more typical, i.e. representative, of the conjunctive concept *pet fish* than either *pet* or *fish* is: this example inspired the name of the cognitive phenomena, the *guppy effect*. Let $\mu(X)$ represent the typicality of *guppy* within category X; then the guppy effect is encountered when

$$\mu(A, B) > \mu(A) \text{ or } \mu(A, B) > \mu(B) \tag{1}$$

or in other words, when *guppy* is more suitable to represent the conjunction of A and B (e.g. *pet fish*) rather than simply A (i.e. *pet*, from the first inequality) or B (i.e. *fish*, from the second inequality).

Hampton [6] identified experimentally an effect similar to the guppy effect, named *overextension*, for the membership weight (which is different from typicality). Estimated weights seem to be ouside the classic polytope defined by classic membership relations [1]. Osherson and Shafir [9] later showed that the guppy effect (or conjunction effect) is highly correlated to the conjunction fallacy, which is a cognitive bias for which subjects estimate more likely the conjunction of two events A and B than only one of them.

Let us now consider the combination of tags in social tagging systems, which is the analogue of the combination of concepts of the previous examples. According to Vander Wal's classification [13], we consider quantitatively the data coming from *broad* folksonomies (such as del.icio.us[4], where many people tag the same item) and not from *narrow* folksonomies (like Flickr, where only few people tag an object), since determine meaning in the relationships between tags might be more challenging in the latter class of folksonomies, due to the low number of users tagging the same object and to inconsistencies amongst tags generated by the use of personal language.

First of all, we evidence an important difference between social tagging systems and cognitive experiments about typicality: in the former, users spontaneously write the most typical tags, while in the latter they are asked to rate the typicality of words or their combination. It follows that in a social tagging system the number of users that tagged the web content x as A will always be higher than the number of users that tagged x as A *and* B: in fact we are studying the number of written tags, not judgements. However, experiments show

[3] Where membership in a set is dichotomous, i.e., either 1 or 0.

[4] `http://www.del.icio.us/`

that the combination of concepts A and B is judged more typical than single concepts (only B for example). In order to overcome this problem, we distinguish between an *implicit* and an *explicit* tag activation process. A web content x explicitly activates a tag when the user writes it in the social tagging system. From a cognitive point of view such activation changes the user's cognitive state: in fact the next tags will be influenced by the first. On the contrary, a tag is supposed to be implicitly activated if it is not written as a tag, but it can help users, with mental associations, to explicitly activate other tags. In typicality experiments, if the combination of two tags is chosen, this means that the two tags are explicitly activated. Conversely, if only one is chosen as typical, then only this was explicitly activated.

Now we are ready to give the first definitions, limiting our analysis to two tags A and B: let $N_x(B)$ be the number of times that the web content x has been tagged as B *and not* as A. This measures the explicit activation of B, without being disturbed by the explicit activation of A. Similarly $N_x(A)$ is the number of times that the web content x has been tagged as A *and not* as B (explicit activation of A). Such quantity corresponds, in typicality experiments, to the number of subjects that choose only one tag as typical.

In the case of double explicit activation, the order becomes important. Thus $N_x(A, B)$ counts the number of times tag A is written before resource x is also tagged with B. Vice versa, $N_x(B, A)$ is the number of times x has been tagged as B and then A. Even if actual social tagging systems do not allow to quantify this *order effect*, it seems to be quite evident in general that $N_x(A, B) \neq N_x(B, A)$. Of course, we can translate such quantities into probabilities, obtaining:

$$\begin{aligned} p_x(A) &= N_x(A)/N \\ p_x(B) &= N_x(B)/N \\ p_x(A, B) &= N_x(A, B)/N \end{aligned}$$

where $N = N_x(A) + N_x(B) + N_x(A, B)$ is the total number of users involved in the experiment. We define also the complementary quantities

$$\begin{aligned} p_x(-A) &= 1 - p_x(A) \\ p_x(-B) &= 1 - p_x(B) \end{aligned}$$

In order to obtain such measures, we define a query in a social tagging system as the search of all the bookmarked pages relevant to a particular topic x that are associated to one or more tags. For example, in del.icio.us such query can be obtained by typing del.icio.us' URL followed by `?p=x+tag:B-tag:A`, where it is important to explicitly exclude tag A, i.e. `-tag:A`, leading to $N_x(B)$. As described before, this represents the situation where users considered the content x more typical as B then A or their combination: in other words, only tag B has been explicitly activated.

The *tagged-guppy* effect can thus be written as

$$p_x(B) < p_x(A, B) \tag{2}$$

where A is the most representative tag (and thus with $p_x(A) > p_x(B)$). As shown by [7], equation (2) is not consistent with classic or fuzzy set theory in the hypothesis that users evaluate tags by considering the features that x shares with tags A and B. In fact, if $S_x(A)$ is a set of features consistent with tag A and $S_x(B)$ a set of features consistent with tag B, then $S_x(A, B)$ is the set of features common to A and B. The number of features in $S_x(A, B)$ is always lower than the number of features in $S_x(B)$ (i.e. intersection of sets), and thus the users should tag x as B with a probability $p_x(B)$ higher than $p_x(A, B)$. The tagged-guppy effect reflects the condition where tag B alone is less representative than the combination of tags A and B. Such effect, in combination with the previously described order effect, i.e. $p_x(A, B) \neq p_x(B, A)$, will find a coherent explanation in the quantum-inspired framework, by means of the concept of contextually.

3 Basic definitions and predictions of the quantum-inspired model

At the moment, the tagged-guppy effect has not been described in other studies, even if it is strictly connected with the guppy effect and the conjunction fallacy. In this paper, we don't consider models for the standard guppy effect since we want to focus on the tagged counterpart of the effect.

Specifically, we describe here the main definitions and predictions of a quantum-inspired model for the tagged-guppy effect, which is based on the quantum formalism used to model the conjunction fallacy [2, 3].

The following postulates, derived from [2] and translated in terms of social tagging systems, can be easily recognised as an adaptation of the basic postulates of orthodox quantum mechanics to cognitive systems. In fact, our idea is to use only the mathematical apparatus of quantum mechanics and the basic concepts of measurements and contextually. Note that in the following we use the Dirac notation to represent vectors and their conjugate transpose. According to this notation, $|A\rangle$ represents a column vector (also called ket), while $(|A\rangle)^\dagger = \langle A|$ is its dual: the conjugate row vector (bra). The inner product between vectors A and B is then written as $\langle A|B\rangle$.

Postulate 1 *The interpretation of a web content x defines a cognitive state represented statistically by a vector $|x\rangle$ that lies within a high dimensional vector space (its dimensionality depends on the number of tags that the web page has activated in the user). We can define for such space a basis, formed by a set of mutually orthogonal and unit length spanning vectors. From a psychological point of view, each basis vector represents a unique combination of tags which can be activated by the web content. The state vector $|x\rangle$ is a unit length vector in this space that represents statistically the actual activation of tags by the web page.*

Postulate 2 *Each tag A is represented by a subspace of the vector space, and each subspace has a projector P_A that is used to evaluate the tag.*

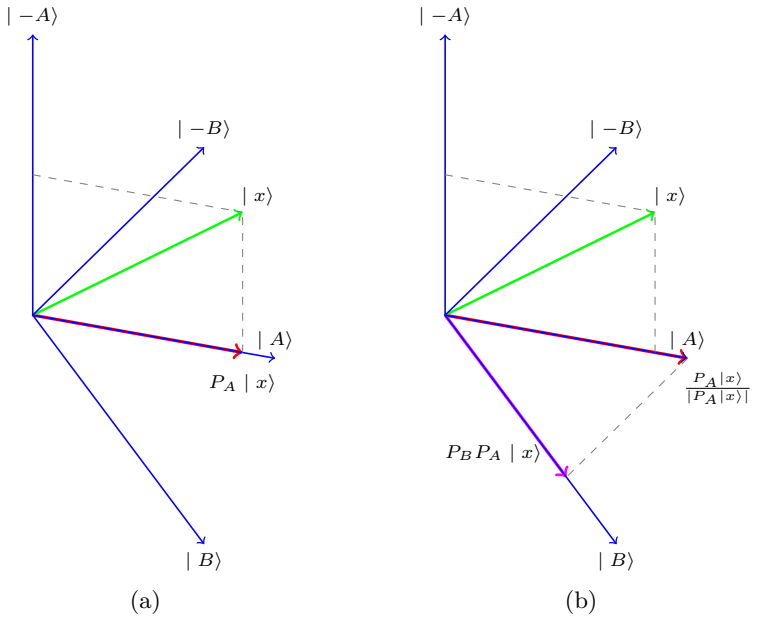

Fig. 1. In Fig.1(a) the web content x is tagged with A, i.e. vector $|x\rangle$ is projected on the basis vector $|A\rangle$. The length of the projection is $P_A|x\rangle$, and thus the probability of tagging x with A is $|P_A|x\rangle|^2$. In Fig.1(b) the projection of vector $|x\rangle$ on the basis vector $|A\rangle$ is rescaled to unitary length, i.e. $\frac{P_A|x\rangle}{|P_A|x\rangle|}$, and it is projected on the basis vector $|B\rangle$ (i.e. x is tagged with B after being tagged with A); the length of the projection on $|B\rangle$ is $P_B P_A|x\rangle$. Thus, the probability of tagging x with A and then B is $|P_B P_A|x\rangle|^2$.

Postulate 3 *The probability of assigning a tag A to a web content x equals to the squared length of the projection of the state vector onto the subspace representing the tag, that is $|P_A|x\rangle|^2$. Such probability is proportional to the number $N_x(A)$ of times that the content x has been tagged as A.*

Postulate 4 *When resource x is tagged as A, the original state vector $|x\rangle$ changes to a new conditional state vector $\frac{P_A|x\rangle}{|P_A|x\rangle|}$, which is the projection onto the subspace representing tag A, but now normalised to have unitary length.*

Note that this postulate has important implications. In fact, after tagging a web content x, the initial vector changes into a different state: tagging a content perturbs the initial cognitive state.

Postulate 5 *If two tags A and B correspond to two projectors P_A and P_B which can be written with a unique common basis, then such tags are said to be compatible. This means that the two projectors are mutually commuting (i.e. $P_A P_B = P_B P_A$). Vice versa, if two tags must be evaluated using projectors relevant to two different bases, then such tags are said to be incompatible. Then*

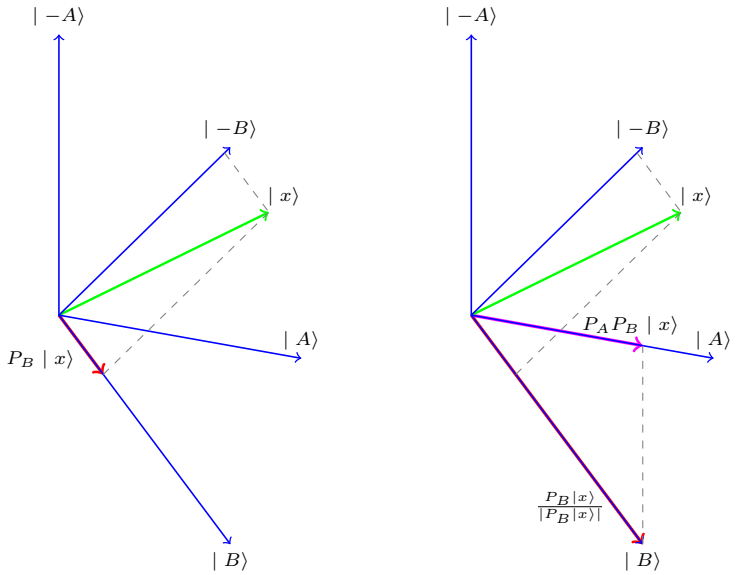

Fig. 2. In Fig.2(a) the web content x is tagged with B, i.e. vector $|x\rangle$ is projected on the basis vector $|B\rangle$. The length of the projection is $P_B|x\rangle$, and thus the probability of tagging x with B is $|P_B|x\rangle|^2$. In Fig.2(b) the projection of vector $|x\rangle$ on the basis vector $|B\rangle$ is rescaled to unitary length, i.e. $\frac{P_B|x\rangle}{|P_B|x\rangle|}$, and it is projected on the basis vector $|A\rangle$ (i.e. x is tagged with A after being tagged with B); the length of the projection on $|A\rangle$ is $P_AP_B|x\rangle$. Thus, the probability of tagging x with B and then A is $|P_AP_B|x\rangle|^2$. Note that the length of $P_AP_B|x\rangle$ in Fig.2(b) is the same of $P_BP_A|x\rangle$ in Fig.1(b). In fact, tags A and B correspond to two projectors which can be written with a unique common basis, i.e. P_A and P_B are compatible (see Postulate 5).

the projectors do not commutate, and the order of evaluation of the tags becomes important.

Compatibility requires using a higher dimensional space to form all combinations, whereas incompatibility can make use of a lower dimensional representation by changing perspectives. Thus, incompatibility provides an efficient and practical means for a cognitive system to deal with all sorts and varieties of questions: such situation is assumed to be normal for web users in social tagging systems, where users perform fast analyses of web contents.

We now focus on the consequences of the previous postulates upon the quantum-inspired model for social tagging systems. The probability that x is tagged as B and not as A (reflecting the situation where people judge x typical as only B) can be written in terms of the projector P_B as

$$p_x(B) = |P_B|x\rangle|^2 \tag{3}$$

Similarly, the probability to tag content x as A and then as B is defined as

$$p_x(A,B) = |P_B P_A|x\rangle|^2 \tag{4}$$

where the first applied projector is P_A and then P_B. Such probability can be expanded in the form $p_x(A,B) = p_x(A)p_x(B|A)$, where $p_x(B|A) = |P_A P_B|^2$ is the conditional probability of B given A (we operationally define it as $p_x(B|A) = N_x(A,B)/N_x(A)$).

The order of evaluation of combined tags is important in the hypothesis of incompatible tags, because the evaluation of the first tag modifies the mental state before evaluating the second. Thus in general we have that $p_x(A,B)$ can be different from $p_x(B,A)$. Quite importantly we assume, as it is done in [2], that when two tags are combined the users first evaluate the most representative tag and then the other. Conventionally we consider A the tag more representative, which means that $p_x(A) > p_x(B)$. The mathematical rules of quantum mechanics can be then used to express $p_x(B)$ in terms of the probability to tag x as A by means of the quantum version of the law of total probability [2]:

$$|P_B|x\rangle|^2 = |P_B P_A|x\rangle|^2 + |P_B P_{-A}|x\rangle|^2 + 2Re\langle x|P_{-A}P_B P_A|x\rangle \tag{5}$$

where the last term is called the *interference term*. This term is analogous to the correlation term between two vectors and, if sufficiently negative, it can determine the presence of the tagged-guppy effect. Specifically, the interference term, responsible of the conjunction fallacy in the quantum-inspired model, fulfils the following equivalence:

$$2Re\langle x|P_{-A}P_B P_A|x\rangle = 2cos(\phi)\sqrt{p_x(A)p_x(-A)p_x(B|A)p_x(B|-A)} \tag{6}$$

where $p_x(-A) = 1 - p_x(A)$ and $p_x(B|-A) = 1 - p_x(B|A)$.

Equation 5 is consistent with the total probability formula used for the conjunction fallacy [2] and can be explained in the following way: the probability that x explicitly activates A and then B can be used to compute the probability of the explicit activation of only B, but we must add a term which describes the interference of all the possible paths implicitly activated from x to B. The interference term vanishes when $p_x(A)$ or $p_x(B|A)$ are very near to zero or to 1. This can be explained by noting that interference is present when there is more than one possible path from x to B.

4 Empirical study

In the following we perform a small scale empirical study. The aim of this study is to evaluate the quality of the predictions of our quantum-inspired model: the study however cannot be considered as an empirical validation of the model due to its small scale nature. A thorough validation and evaluation of our model will be subject of future work.

To perform the empirical study, a number of tags with the associated statistics have been drawn from del.icio.us in April 2010. These tags have been selected

so that they exhibit the presence of the tagged guppy effect. In general, it is not difficult to provide examples of tagged-guppy effect: it is sufficient, given a topic x, to find a more representative tag A and another less representative B with a non-null overlap with the previous. For example, the clustering service of Flickr helps to find such pairs of tags. We also tried to use the experimental data reported by Hampton in [6]. However, since we are working on a broad folksonomy, many of the categories in Hampton's experiment, if used as tags for a particular item, do not have associated pages. This is likely to be because those words are not useful as tags for the users. Nevertheless, in general, when the categories used in [6] are meaningful also as tags, we can observe a good agreement between membership-weight experiments and tests on folksonomies.

Once tags and associated statistics are extracted, we compute the value of $p_x(B)$ that is predicted by our quantum-inspired model. In Table 1 we report the prediction of our model, $p_x^Q(B)$, obtained using Equation 5. The cosine of the angle ϕ can be regarded as a free parameter, which could be potentially estimated for any experiment in order to fit accurately the empirical values. However, we expect that the interference term resulting in our experiment is a mean value over a large set of subjects and thus it is unlikely to reach its extremal value -1. To this end, we set $cos(\phi)$ to -0.75, as an attempt to approximately fit the real data.

We compare the prediction of the model against the actual value of $p_x(B)$ obtained from the del.icio.us statistics. In the table, the tagged-guppy effect is highlighted in bold, and it has an increasing strength from the first row towards the last one (the strength of the tagged-guppy effect can be measured as the difference $p_x(A, B) - p_x(B)$). From the reported results, we can note that the quantum-inspired model, although quite simplified (since it does not consider the other tags involved), captures the presence or absence of the tagged-guppy effect and predicts quite correctly $p_x(B)$ given $p_x(A)$ and $p_x(B|A)$.

x	$A - B$	$p_x(A)$	$p_x(B\|A)$	$p_x(A, B)$	$p_x(B)$	$p_x^Q(B)$
Work	Art - Office	0.64	0.12	0.08	0.28	0.20
Work	Art - Desk	0.96	0.01	0.01	0.03	0.02
Work	Design - Desk	0.98	0.01	0.01	0.01	0.01
Work	Office - Desk	0.93	0.04	**0.04**	**0.03**	**0.04**
Work	Tool - Idea	0.73	0.20	**0.15**	**0.12**	**0.14**
Adobe	Design -Software	0.49	0.60	**0.25**	**0.20**	**0.17**
Parrot	Bird - Pet	0.72	0.25	**0.18**	**0.10**	**0.13**
Work	Computer - Mac	0.62	0.40	**0.25**	**0.14**	**0.16**

Table 1. For different combinations of argument x and tags A, B we show respectively the real frequencies of tags A (third column), B *given* A (fourth column), A *and* B (fifth column), B (sixth column), and the value of B predicted by our model, i.e. $p_x^Q(B)$ (seventh column). The presence of the tagged-guppy effect is highlighted in bold.

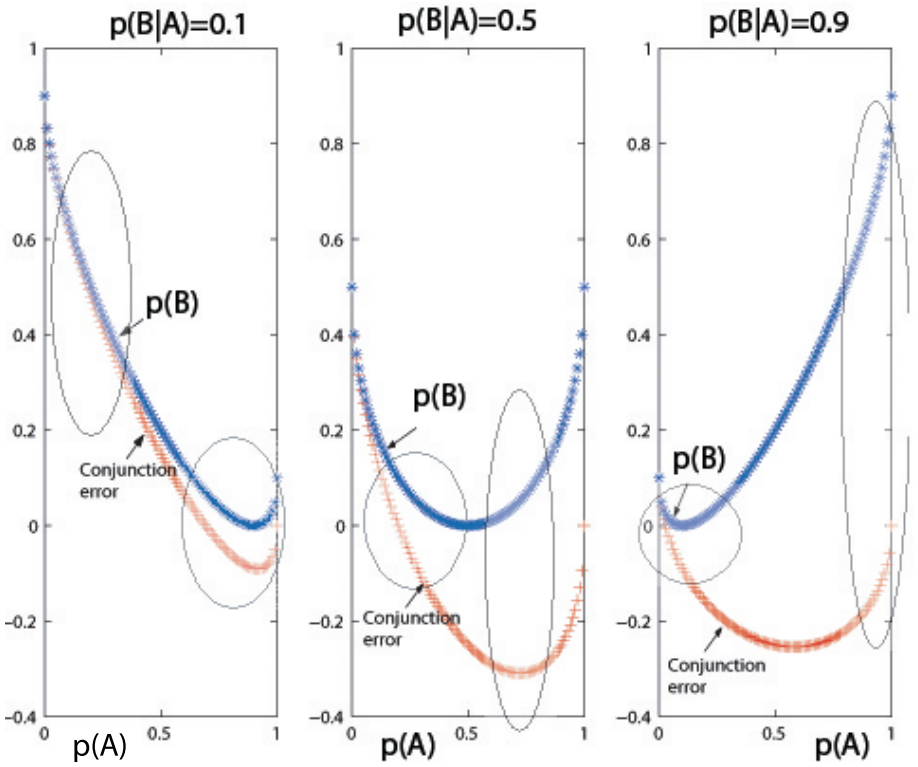

Fig. 3. Maximal conjunction errors for three different values of $p(B|A)$.

The complete quantum-inspired model for the conjunction fallacy [3] presents other cases, as shown in Figure 3 (for example when the conditional probability $P(B|A)$ is very high); but since these cases rarely occur in broad folksonomies, we do not consider them in the present paper.

Figure 3 also evidences that the fallacy is present in the following particular combinations of $p_x(B|A)$ and $p_x(A)$:

(a) $p_x(B|A) < 0.2$ and $p_x(A) > 0.7$;
(b) $p_x(B|A) \in [0.2, 0.7]$ and $p_x(A) > 0.3$; or
(c) $p_x(B|A) > 0.1$ and $p_x(A) > 0.01$

These situations are consistent with the data reported in Table 1, where the comparison between the experimental value of $p_x(B)$ and the computed $p_x^Q(B)$, i.e. the value of $p_x(B)$ predicted by our quantum model, evidences good agreement. A more precise match can however be obtained by adjusting the value of $cos(\phi)$, which represents the intensity of the interference effect.

Finally, we note that the order effect defined in section 2 is consistent with postulate 5: projectors relevant to tag A and B do not commute, and thus $p_x(A, B) \neq p_x(B, A)$. Unfortunately, the web tools available for social tagging

systems do not allow to quantify and test such effect. However, it is sufficient to search for bookmarked pages with tags A (more representative) and B (less representative) to notice that in general the order (A, B) is more likely than the order (B, A). From a quantum-inspired point of view, this can be explained by the fact that the projection of P_B on $|x\rangle$ has a low probability, and the following projection on P_A further reduces the final probability. Conversely, when we consider the opposite tagging order, the initial projection on P_A is more likely, thus resulting in the transition to the second tag B with a higher probability. From the point of view of classical set theory, the interpretation of such effect in terms of common features is incompatible with the experimental results: in fact, the number of common features of A and B is independent from the tagging order.

5 Conclusions

In this paper we have presented a version of the guppy effect for the social tagging systems. In particular, we have provided a quantum-inspired description of the cognitive processes involved, evidencing the presence of interference effect in the combination of tags.

We have shown empirical data, drawn from the del.icio.us tagging system, evidencing a good agreement with our model, even if a complete crawling of the Web service has not been produced. According to our model, the combination of tags results to be more representative than the single tags.

Moreover, some new predictions can be derived from our quantum-inspired model: (i) the order of typing tags is useful to better understand cognitive processes, even if at the moment this can not be tested, (ii) an analysis of the original quantum-inspired model for the conjunction fallacy, which considers a wider set of combinations of representative/non-representative tags, may also provide new predictions.

The quantum-inspired model that we have proposed in this paper is the first step towards a more comprehensive model of the combination of tags in folksonomies. Future avenues will be directed towards:

- the validation of the model via a thorough empirical investigation of tags combinations in broad folksonomies as del.icio.us;
- the consideration of higher dimensional subspaces for representing tags, which would allow to evidence non symmetric overlaps when combining tags
- the use of the Grover's algorithm, which has been already adopted to describe cognitive processes [4] relevant to memory and probability estimation, for a deeper study of the cognitive heuristics.

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