The Geometry of Novelty and Diversity

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Summary

Given an exhaustive set of elementary building blocks, we should be able to conceive, in principle, a state space where all possible configurations exist (past, present and future; feasible as well as not feasible). In this space, novelty corresponds to locating specific configurations not yet actuated and diversity corresponds to maintaining a set of configurations geometrically far from one another. In this paper we discuss whether this representation vaguely resembles a processes which may generate novelty and diversity in Nature and whether this representation is useful, both conceptually and pragmatically. We suggest that a dynamical, rather than static, view of state space, not only encompassing self-organisation, but itself self-organising, is needed to capture our intuitive notion of novelty and diversity. The analysis raises the concern, already proposed by Kauffman, that computation in such space may not be possible, leading to question the use of computer modelling as a tool to understand the generation of novelty and diversity.

Keywords: Self-Organisation, Computation, Diversity, Novelty generation, Modelling.

The hypothesis

"*You really understand an algorithm when you've programmed it*" (**Error! Bookmark not defined.** 1997). This statement seems reasonable. We think about a problem, devise a path to solve it and formulate it as a sequence of steps (algorithm). Alternatively, someone may tell us how to solve it and give us the sequence of steps, which we may or may not fully understand. The real test lies in coding the algorithm. First, by writing the code, we realize whether we really understood the instructions and whether the sequence is complete. Second, by running the code, we verify the algorithm was properly written, does what we intended to and ultimately whether it solves the problem. Every programmer is familiar with this process.

I slightly paraphrase Chaitin's statement into "*You really understand a process when you can model it*". At first sight the analogy seems fairly innocuous. It plays on the correspondence between and algorithm (the computer model) and a process. Looking deeper though, I have taken a gamble and highlighting the gamble and the risky consequences is the purpose of this paper.

Assuming for a moment that the analogy is correct, then it follows that "*You really understand the generation of novelty and diversity when you can model it*". This step is safe, since it simply implies that the generation of novelty and diversity is a process.

Given the last statement, we need to ask:

1) Can we model the generation of novelty and diversity?
2) If yes, what kind of state space (what kind of virtual world) do we need to work on?
3) If not, is this merely a current limitation which can be overcome, or is it a fundamental limit? Can it be modeled at all?

Borges' Library and "classical" state spaces.

In his short story "The Library of Babel", Borges describes an imaginary world consisting of an immense library (**Error! Bookmark not defined.**). The library contains a vast number of similar hexagonal rooms in which "*there are five shelves for each of the hexagon's walls; each shelf contains thirty-five books of uniform format; each book is of four hundred and ten pages; each page, of forty lines, each line, of some eighty letters.*" The letters in each book form apparently structure-less strings. The Library is inhabited by Librarians, who, generation after generation, have tried to decipher the writings within the books, in order to understand the meaning of the library (their world), as well as its history and geometry.

Here, we employ this imaginary library as a starting point for our discussion because of one noticeable mathematical property: the library represents a state space. Consider a space of 1,312,000 dimensions (one for each letter in a book), whose axis have length 25 ("*the orthographical symbols are twenty-five in number").* Each book in the library represents a point in the state space and the totality of the books represents an exhaustive coverage of this space ("*.. the Library is total and its shelves register all the possible combinations of the twenty-odd orthographical symbols*"). A pictorial representation of this state space (limited to 3D) is given in Figure 1.

The size of the state space is finite, but so large, that for all possible applications to human affairs it may very well be considered infinite. All things which can be written and communicated within the length of a book are contained in this space: "*the archangels' autobiographies, the faithful catalogues of the Library, thousands and thousands of false catalogues…. the true story of your death, the translation of every book in all languages* " as well as yesterday's conversation in your favourite pub. More important for our discussion, "*the minutely detailed history of the future*" and tomorrow's conversation in your favourite club are also contained there. The space contains everything which was said, everything which will be said as well as everything which *could* be said but never will.

Figure 1. Schematic representation of Borges' State Space. Each point in the state space represents a book.

I will use this library as example of a 'classical'[1] state space because its relation to writing and human discourse is natural to all readers, independently of their background; however analogous spaces could be created by employing musical notes (thereby containing all possible melodies), genes in chromosomes, components in a mechanical engine, delayed-coordinates in dynamical systems etc..

Three features of this state space are particularly relevant to this discussion. First, it contains everything *a priori*. Whether a string is actuated in the real world may depend on human dynamics, chaos, chance or fate; but all strings exist, *potentially*, since the very moment the state is conceived. Second, the state space is *static*; there is no provision for change in its size, dimension or topology. Third, the state is devoid of meaning; strings are just collections of symbols. The Librarians interpret the strings and give them meaning, when possible, for their own purpose, but the meaning is not inherent in the state space since the Librarians are not part of this abstract space[2], but are external to it. This point is crucial, so I emphasise it via Figure 2.

Here I want to address three questions:

1) What do novelty and diversity look like in this space?
2) How does the space look like to the Librarians?
3) Who/what generates novelty and diversity? And who is it relevant for?

What do novelty and diversity look like in this space? Generating novelty, in this scenario, equates to locating (purposefully or by chance) strings which differ considerably from the strings so far actuated. Because the strings do not have any implicit meaning, string difference reduces to distance in the space, which in turns

[1] I call this space 'classical' because this is the most common description of state spaces in the physical sciences.
[2] Notice the difference between the (imaginary) physical Library, where the Librarians live, and the abstract state space it represents, that is the collection of symbol strings, to which the Librarians are 'external'.

depends on the difference in symbols at each position along the string[3]. Similarly, diversity implies that a number of strings sufficiently distant from one another are 'actuated' at the same time.

Figure 2. A librarian (left) can study the state space represented by Borges' Library, but is necessary external to it. Arcimboldo's imagination (right) depicts what a librarian *would* look like if he was part of the state space.

How does the space look like to the Librarians? The only way a Librarian can access the state space is by reading a book, and necessarily a single one at the time. A Librarian can not see the 'landscape' of the space, only the local information contained within each book. The state space is immense and made up mostly of structure-less strings, that is, by strings which appear random ("*for every sensible line of straightforward statement, there are leagues of senseless cacophonies, verbal jumbles and incoherences*"). Finding an apparently non random (thus possibly meaningful) string is very unlikely, and Borges described very vividly the Librarians' desperation in their search. Worse still, a book does not carry any information about adjacent books or books within the same room, that is, the space has no structure. This means that, if a Librarian, by incredible strike of luck, finds a meaningful string, he will not have any information on how to find another one[4]. In optimisation jargon, the space is a rough, structure-less plateau, punctuated by isolated peaks representing meaningful strings; basically a small set of needles in an immense haystack, as described in Figure 3.

Who/what generates novelty and diversity? And who is it relevant for? It is normally assumed that 'successful' configurations (whether novel or not) are found in Nature (or at last in the biosphere[5]) via the process of evolution, which is usually viewed as a search (without any anthropological sense of purpose) in a state space of configurations[6]. Can this search actually be carried out in the Library? First, we would

[3] Recall that strings (books) are just points in the state space, whose difference needs thus to be seen as distance between the corresponding points. If we use standard Euclidean metrics, this depends on their coordinates, that is on the difference in symbols at each axis (location in the string) since this is how the state space has been constructed.
[4] Changing a handful of symbols will make a meaningful string meaningless. So in order to move from one meaningful string to another, a large set of symbols need to be changed together in coordinated fashion; another very unlikely possibility.
[5] We borrow this term from Kauffman to refer to the part of Nature which includes living organisms as their effects on it.
[6] This is the underlying idea behind Genetic Algorithms used in numerical optimisation.

need some agents to carry out the search (the Librarians), who, as we saw, are not part of the space. Second, the agents need to be given a target to search for (a purpose or a meaning). This also is not contained in the space; it is the Librarians who decide to search for the "*the origin of the Library and of time*", this is not an implicit aim inherent in the Library. Consequently, novelty and diversity can not be searched either; in order for a book to look 'novel' to a Librarian, it has to have some sort of meaning, but the space is devoid of meaning. Finally, no new book can be generated because everything is given a priori.

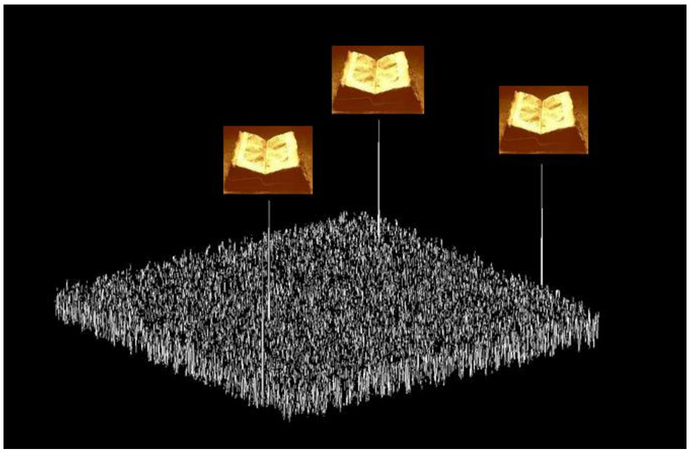

Figure 3. 'Readable' book are like needles in an immense haystack of meaningless strings of symbols.

We then need to conclude that agents, purpose, meaning, novelty and diversity are all externally given, they do not exist in Borges' space, in which change is also impossible. This suggests that a state space representation of this kind may not be suitable to study the generation of novelty and diversity. Still, this is probably the most common use of the concept of state space in science. It is perhaps not surprising that supporters of Intelligent Design and Creationism have relied on the unlikely nature and unlikely success rate of a blind random search in such structure-less, meaningless and immense space to deny the validity of scientific theory of evolution. Without any need for divine intervention, we may very well suppose that Nature works in more 'clever' abstract spaces.

It is fairly simple to understand why Borges' state space is not suitable to represent the range of human writing and discourse[7]. Orthographic symbols are not the 'units' of human thinking. Indeed written characters have been invented not only much after humans started to think and speak, but also well after they starter to write. Characters are a useful tool to annotate and communicate thoughts by representing phonemes, but they only indirectly relate to thoughts. A better state space is needed for this purpose, as well as to investigate novelty and diversity.

Boden's state space

In addressing human creativity, Boden discusses a different kind of abstract state space, in which different configurations of human artefacts (music, architecture,

[7] Obviously this was never Borges' aim anyway. We call this Borges' spaces simply as a label and for ease of discussion in this work.

literature, scientific theories) may exist. Boden's state space differs from Borges' in two main features:

1) the axis of the space are closer to the 'units' humans use for thinking and creating, that is, they are better building blocks. Taking poetry as an example, the axis may represent words, ideas, analogies, images, metrics or rhythms. These units have a more direct meaning in the context of what the space wants to represent. Figure 4 represents one such state.
2) Boden introduces the concept of constraint and emphasises its importance. Novelty and diversity, she correctly argues, have to do with exploring as much as with limiting what is possible. Without constraints, without symmetry breaking, when everything is possible, only random noise is likely generated, as we saw in Borges' space.

Figure 4. Example of Boden's state (sub)space. The axes represent 'concepts or ideas'. By mixing the idea of a building, a sailing boat and the vicinity to a coats line, the Sydney Opera House can be conceived.

Boden's state space is inhabited by Artists, or other creative souls. Although still very large, Boden's space is immensely smaller than Borges'. Also, the ratio of meaningful-to-meaningless configurations is now much higher, since the building blocks themselves now posses some sort of intrinsic structure. Searching the space is now much easier and it is now not unconceivable that chance (in the form of a random action, but not necessary limited to it) may help generating meaningful and novel configurations. More importantly, the state space also has now some sort of underlying structure. Discovering a novel configuration (by deleting/adding or moving a constraint, for example) would potentially enable an Artist to understand how to generate more instances of novelty, by searching nearby regions or by porting the modification (by analogy) to another constraint.

Nevertheless, even Boden's state space is devoid of agents and consequently of meaning and purpose for any sort of search. The Artists of Boden's discussion search the space, but are not part of it; they live outside it, as described in Figure 5.

This leads to a number of observations:

1) in order to collapse the large number of dimensions in Borges' space into the axis of relevant structures in Boden's space a considerable amount of information is needed. Who provided this information? How was it generated? Clearly the Artists detect and choose the 'units' of human thinking, so structure is again externally generated.
2) By changing the 'axis' Artists can change the space, thus the space is not static. However, change is also external.
3) Who provides the purpose for the search? Once again, it is the Artists who decide what novel artistic work is worth pursuing.

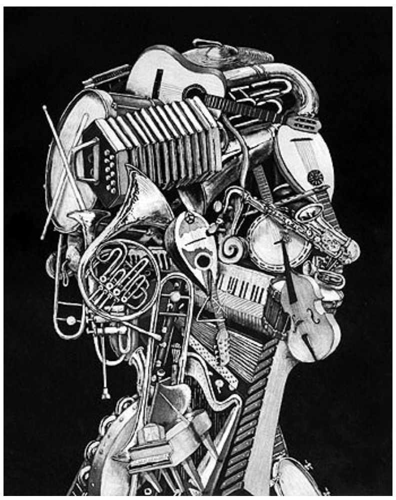

Figure 5. What the inventor of musical instruments would look like in Boden's space if he was part of the space.

Once again, all features which directly relate to the study and generation of novelty and diversity are externally given and can not arise spontaneously in the state space.

Kauffman's state space

In his pioneering work 'Investigations' (**Error! Bookmark not defined.**), Kauffman specifically discusses what a state space for a biological problem may look like. The main points can be summarised as follows:

1) the way we do science, whether by mathematical modelling or by equation solving, requires (a) to state *a priori* initial and boundary conditions, and (b) to follow a set of rules (apply physical laws to the initial conditions) until an outcome is produced. Kauffman questions whether we can actually define *a priori* ('pre-state' using his words) the initial and boundary conditions for

biological systems. Without formal proofs, he conjectures that, in practice, this is not possible. It then follows that 'standard' science can not be carried out in sufficiently complex biological problems. Turning to our state space, this means that the shape, size and topology of the state space can not be determined a priori. It also means that, unlike previous examples, we can not determine a priori all possible (past and future) configurations. Said differently, not everything which can be actuated potentially exists from (our) very conception of the state space.

2) Since living beings interact with the physical world and they modify it, they do not simply inhabit this space (like Borges' Librarians or Boden's Artists), but they are an integral part of it. The agents are now *not* external to the space, but are entities (configurations) *of* the space itself. In the following we will simply call them Living Beings.
3) Because Living Beings are happy to live[8], their actions (eat, reproduce, flee predators, compete, collaborate, etc.) now have an aim in terms of survival. This aim thus provides a *meaning* to things and actions in the space (tastes good/bad, danger/opportunity, mate/flee, etc.). Importantly, since the Living Beings are part of the state space, it follows that meaning in Kaufman's space is not externally given, but it arises internally by the actions of its agents.

We thus obtain a state space very much unlike the ones previously discussed. First, it has an *internally* generated meaning in the basic need of the Living Beings to survive. Thus the search for successful (and potentially novel) configurations in the space is not guided by external principles. Second, since the search is an intrinsic part of the agents survival skill, the search itself 'provides' for (that is, evolves) agents who are skilled at searching. These agents are skilled at detecting and processing the features in the state space which are more informative about the space (their environment) internal structure. Third, the space is not static, rather it changes and is changed by the agents which inhabits it. The way this state changes shape and evolves is beautifully described by Kauffman in his entertaining tale of Gertrude, the squirrel which happens to develop a unnecessary flap of skin, with no apparent functionality, until, by chance, chased by a predator, it allows her to glide, survive and, eventually, discover a novel concept: flying (**Error! Bookmark not defined.**, Chapter 6). This concept could not exist before the chance discovery, because the unnecessary flap of skin did not exist and its potential use could not be envisaged. We could claim that, in principle, the unnecessary flap of skin, as well as the chaser, Gertrude and the tree are all made up of the basic building blocks, which, in principle, pre-existed the occurrence of the discovery of flying, as in Borges' space. But this view is not useful, and surely not realistic, when the Living Beings are not merely external observers, like the Librarians in Borges' Library, rather are integral and active components of the space itself. For them, the space is not a static set of items there to discover, rather is a plastic environment which they themselves can mould. Clearly, they could not pre-state all the potential consequences on their existence, their own interaction and interference with the state space. Their being *agents*, rather than merely *observers*, profoundly changes the state space they live in.

This scenario seems to provide for processes which are closer to our intuitive understanding of novelty, that is of objects/actions/concepts which not only could not

[8] Kauffman obviously proposes a more precise definition of life.

be pre-envisaged, not only are different, but are so within a context and a meaning, not just as an arbitrary defined measure of distance.

The key words here (missing in the previous state spaces) are *interaction* and *organisation*. Agents interact with other agents and other entities in the space. They modify them and are modified by them. The resulting feedback loops can generate inter-dependencies, in the form of organised structures, which in turns can dampen or reinforce certain configurations in the state space, whereby certain regions of the (forever changing) state space can become less accessible while others can be more extensively searched.

This dynamical view of a state space, according to Kauffman, comes at the cost of computability, that is at the cost of having to give up our hope to model the process. It is so because traditional science, Kauffman says, can not be carried out within this (forever changing) scenario. Neither can we model the generation of meaning, since this too, arises spontaneously from the dynamics of the state space[9].

This is probably not surprising to non physical scientists, who have traditionally been very sceptical about ever being able to numerically model complex biological processes. But after all it should not be alien to physical scientists either, because of its close analogy to the computational challenges behind modelling chaotic systems. It is widely known that chaotic processes can not be modelled accurately because we will never be able to obtain information of infinite precision about their initial conditions and even the minimal numerical discrepancy will be exponentially amplified by the internal dynamics. Similarly, Kauffman tells us that we will never be able to precisely account for each single element of the initial conditions which may possibly, in the future, interact with the process we care about. It is perfectly reasonable that in modelling Gertrude's world since its very conception, we may have been unable to account for the progenitors which eventually procreated Gertrude's predator, as well as for the insect which happened to drop the seed which developed into the very tree Gertrude happened to be when the predator chased her, exactly at the time the unnecessary section of skip had by chance been developed (and not suppressed by evolution). These (the predator progenitors, the insect, the seed) can be seen as extra dimensions of the state space, or extra sections of some axis, which we could not possibly include in our model of Gertrude's world and which, eventually, interacted with it to the point of creating something as noticeable, novel and unpredictable as flying (see Figure 6).

The effectively uncomputable nature of the processes arising in Kauffman's state space has recently been formally proved in a set of papers (**Error! Bookmark not defined.**; **Error! Bookmark not defined.**; **Error! Bookmark not defined.**; **Error! Bookmark not defined.**; **Error! Bookmark not defined.**; **Error! Bookmark not defined.**; **Error! Bookmark not defined.**) which show how agents interacting with an environment can potentially display computational power which goes beyond what can be achieved via classical computer simulation.

One question still remains open. While Kauffman space allows for the arising of spontaneous and genuine novelty, we may ask what will actually look novel to the

[9] Penrose too claims that meaning is fundamentally uncomputable (**Error! Bookmark not defined.**).

Living Beings. After all, if not everything is defined a priori, and if change is inherent in Kauffman's space, the Living Being should be surrounded by endless generation of novelty and diversity. What will then be classified as novel and what not?

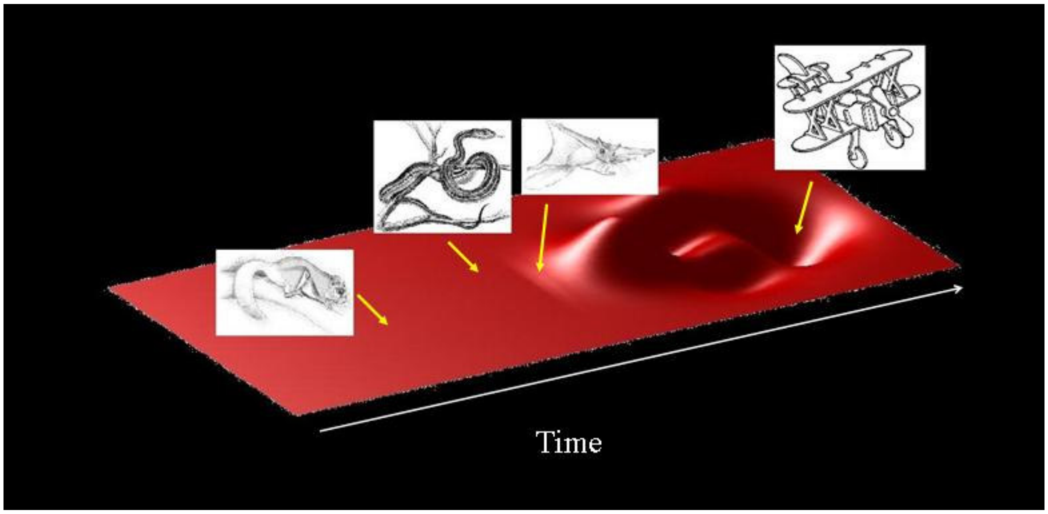

Figure 6. Caricature view of Kauffman state space. Time progresses from left to right. Initially the space is nD (n=2), but when by change Gertrude discovers flight, a new dimension becomes available to the state.

Crutchfield's agents. The question of what a Living Being can see of Kauffman's space thus is addressed by Crutchfield (**Error! Bookmark not defined.**). By employing information theory within an evolutionary context, Crutchfield describes very effectively the process an agent undertakes in order to 'understand' its surroundings. 'Understanding' here means to build a model of how the environment works and use such model to predict, as well as possible, its behaviour. A good model is one which predicts well, is fairly general and fairly small. A model which predicts well and is general allows the agent to adapt successfully to its environment (flee predators, locate food, etc.), while a small model allows for resources to be devoted to other activities (fleeing, eating, etc.). An agent does not have any a priori knowledge of the environment. It can acquire such information via its sensors which collects data from the outside world and can test its model by interacting with it. Darwinian selection provides the ultimate 'result' of the test. The acquired knowledge (model) can then be stored in the agent's genetic make-up (in the form of the agent's structure which survived and adapted to the environment) and then transmitted to its off-springs. Cultural transmission also applies in the case of more complex agents.

The crucial component of this discussion is how the knowledge is represented within the agent, that is, how the model is built. Crutchfield conjectures about the existence of a hierarchy of languages of increasing complexity which are used to make sense of the world. Basic agents will most likely develop a basic language. Only when the basic language is faced with features of the environment it can not model accurately, a more complex language is 'discovered' by evolution, thereby increasing the agent's modelling capabilities.

Multiple levels of Novelty. If we accept this view, then Kauffman's space is empowered with three different levels of novelty. The first one was described above via Gertrude's biography, and shows how new structures and processes arise within the space. I call this *process* novelty. The second is the one we are probably most familiar with, which is the sense of *surprise* which arises in an agent when it faces a process it is not able to model (and thus predict or expect). The third is the novelty in the *representation* of the world which arises when an old model is deemed ineffective and a new model is generated. Finally, the *representation* novelty, which provides new information processes capability, is likely to result in novel *behaviours* in the agents. We can thus imagine that the arising of an instance of *process* novelty may generate *surprise* novelty in an agent. This, undergoing evolutionary pressure, may result in a *representation* novelty. The new *behaviour* the agents is now empowered may provide for a new interaction with the environment which was not possible before. This may generate new *process* novelty, closing the loop and effectively generating a potentially continuous and self-sustaining process, as sketched in Figure 7.

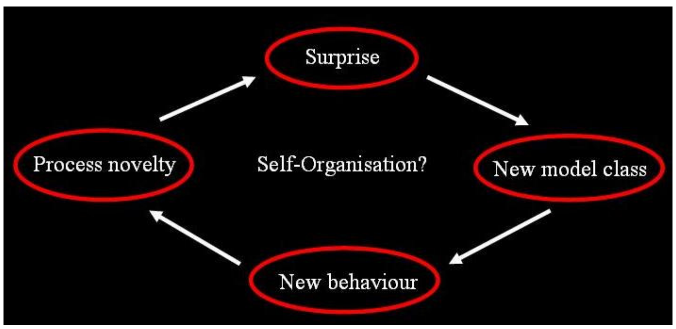

Figure 7. Schematic representation of the levels of novelty in Kauffman's space and their relation.

We have seen that the Living Beings (Crutchfield's agents) are part of Kauffman's state space. By trying to model their environment, the agents have generated a *new space*, a representation space, where different models and languages exist which allow the agents to 'understand' their world. The question arises whether the agents are inside or outside such representation space. It is conceivable that the Living Beings are to the representation space what the Artists are to Boden's space as graphically described in Figure 8. This leads to a final consideration.

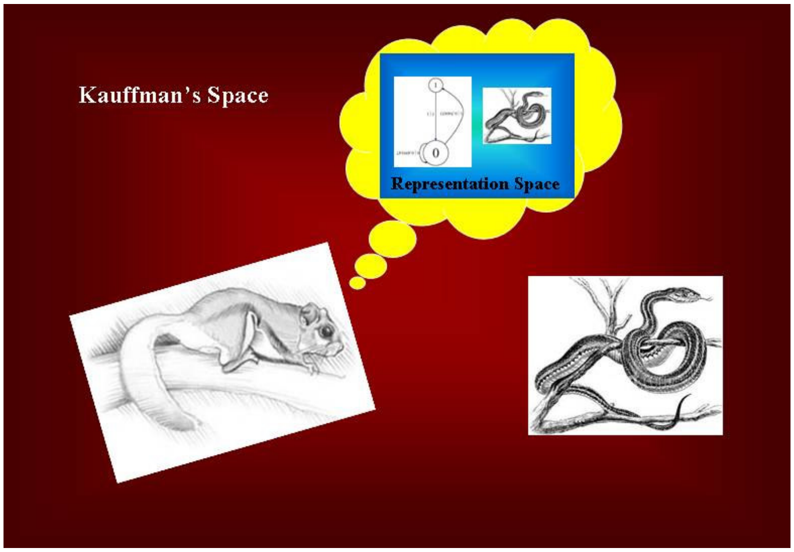

Figure 8. Caricature view of a Crutchfield's agent in a Kauffman's space. The agent builds a representation of its environment, which is located within the agent's representational space.

In a cornerstone of modern mathematics, Gödel proved that given a formal system, that is a set of a priori truths and a set of rules operating on these truths, there will always be statements (theorems/laws) which are true, but can not be deduced by using the very same rules of the system **Error! Bookmark not defined.**. Crutchfield's languages are computational tools, and consequently they are equivalent to a formal system. An agent's model (a single model level in Crutchfield's language hierarchy) thus is constrained by the limitations of Gödel's theorem. The agent's logical tools (the way it model the world, does maths, science, etc.) thus coincide with the transformation rules the logical system is provided with. Given enough time and patience the agent will be able to deduce a large number of truths (deduce many physical laws, for example). Of course it will not be able to deduce the laws which can not be proved within the system (which can not be modelled by its internal model). More important, it will not even know those rules exist nor see them. Now suppose the agent acquires a new piece of knowledge (detects new data via its sensors) or is empowered with a new transformation rule. Suddenly it will be able to deduce (or at least know or see) some truths which were not accessible before. This is because it stepped outside the logical system into a new, larger and more powerful one which includes it. This new system also contains new truths which are not accessible to the agent, but some of the old inaccessible truths are now available. In his work, Crutchfield does not explain how agents may jump from one computational model to the next. If the description I gave of how this may happen (cast in terms of formal logic) is correct, then we have to deduce that the process is not computable. *Novelty then seems to represent a discontinuity dividing levels which are in isolation computable* (Figure 9).

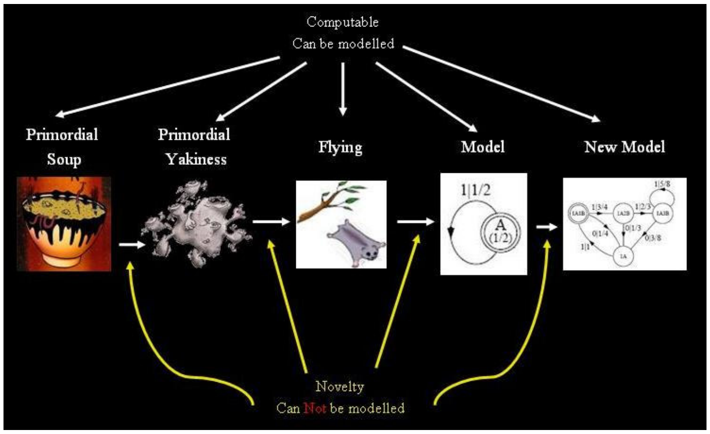

Figure 9. Novelty generation as a discontinuity separating levels of different modelling potential.

"*You really understand the generation of novelty and diversity when you can model it*". Since the discussion presented in this paper faces uncomputability at two levels (Kauffman space and representation space) we are then left to question whether the generation of novelty and diversity can today be really understood.

Discussion

Complex system scientists (like many other scientists) today mostly do science on a computer. We run and design experiments, we test our ideas and we build models of our understanding of reality on a computer. The underlying rationale (hope) is that the world we build in the computer has some reasonable similarities to the real one, at least for the processes we are interested in. Nowhere this assumption can be challenged more than by studying biological processes and in particular self-organisation, adaptation, evolution and generation of novelty and diversity. Not only we do not understand these processes well but we are not even sure whether they are amenable to computer simulation in the first place. The way we do mathematics, so deeply entrusted into the concept of *equation* (that is things being equal), and the way we compute, so deeply entrusted on a set of a priori defined fixed rules, does not seem to leave any opening for the novelty and diversity which we see so obvious and ubiquitous in Nature.

In his book Kauffman suggests that something is wrong with the way we do science, and in this discussion I tried to give a graphic picture of why this may be the case. I tried to show that the most basic and widespread conception of a state space, Borges' space, is at least not useful (if not plainly deceiving) to understand the arising of novelty and diversity in Nature. If we agree with this statement, then we need to be very careful in entrusting computation as our main (if not only) mean to investigate and experiment with these concepts.

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