Geophysical feature removal by multiscale edge suppression

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Summary.

We present a procedure to remove or isolate specific features from 1D profiles or images. The procedure works by manipulating the multiscale edges and reconstructing the image. A modification of the Mallat and Zhong reconstruction algorithm is proposed.

Introduction.

The analysis of 1D profiles and 2D images is common in many disciplines, including geosciences and medical applications. Often the analysis is carried out visually by trained interpreters. A number of filters are available to sharpen, denoise or enhance the data in order to facilitate the interpretation. Such filters are often implemented in the Fourier domain. Hence, they are not local, acting on all features simultaneously.

Mallat and Zhong (1992) proposed a technique to process profiles and images based on multiscale edges. The idea is that the multiscale edges contain approximately the same information as the original image, i.e., the original image can be reconstructed from the information stored in the location and amplitude of the multiscale edges. By manipulating the edges and then reconstructing the image certain kinds of processing are possible. Mallat and Zhong (1992) show examples of denoising by suppressing small edges, statistically related to noise. Lu et al. (1993) show that by increasing the amplitude of the multiscale edges grey-scale contrasts can be enhanced, greatly improving the interpretability of images. However, the characteristic advantage of this technique is that it can be applied to specific areas of the images thanks to the local, or approximately local, properties of the multiscale edges. This is in contrast to conventional Fourier domain approaches.

Often there is the need to remove or isolate specific features from a image. For example we may want to remove large features due to known errors in the data or to isolate specific features for further analysis. This task is not trivial. First we want to avoid subjectivity in the decision on feature localisation. Second we want the image to be smooth around the selected feature. Third we want to automate the process. Clearly such an operation can be performed only through a parameterisation with good localisation properties.

Here we present a technique to remove or isolate individual features based on removal of multiscale edges and subsequent reconstruction. Examples are given for 1D profiles in geophysical applications.

Method.

Our applications are in the analysis of gravity and magnetic images for geophysical exploration. We use a wavelet derived from the same physics that generates the signal under analysis.

Let us consider the magnitude of the gravitational vertical acceleration

$$f_{z_0}(x,y) = -g_z = \frac{\partial V}{\partial z} =$$

$$G \int_{\Re^2} dx'\, dy' \int_{-\infty}^{0} \frac{\rho(x',y',z')(z_0 - z')\, dz'}{((x-x')^2 + (y-y')^2 + (z_0 - z')^2)^{3/2}}$$

where *gz* is the vertical component of gravity, *V* is the gravitational potential, *G* is the gravitational constant, ρ is the density distribution, *x*,*y*, and *z0* are the measurement location and *x'*, *y'* and *z'* the gravity source location.

The Green's function for the gravitational vertical acceleration is:

$$g(x,y,z) = \frac{z}{(x^2 + y^2 + z^2)^{3/2}}$$

The integral of γ over the x and y plane, divided by 2π is unity for all z > 0. Hence, an admissible scaling function is given by $\theta^{(sz)}(x,y) = \gamma(x,y,sz)/2\pi = \frac{1}{s^2}\theta^{(z)}(\frac{x}{s},\frac{y}{s})$.

The wavelet component functions $\psi^{x(z)} = \frac{\partial\theta^{(z)}}{\partial x}$ and $\psi^{y(z)} = \frac{\partial\theta^{(z)}}{\partial y}$ share the property $\psi_s^{i(z)}(x,y) = \frac{1}{s^2}\psi^{i(z)}(\frac{x}{s},\frac{y}{s})$

Thus we find a set of self consistent dilation equations which can be employed as wavelets.

Following this construction, the wavelet transform is, except for a scale factor, the (vector valued) first derivative of the signal smoothed at the scale s. The local maxima of the wavelet transform thus correspond to rapid variations in the image, and are collectively called multiscale edges.

The use of such 'potential field' wavelets gives specific advantages in geophysical applications and allows, under favourable conditions, the estimation of the nature of the signal source. A detailed description of the underlying theory can be found in Hornby et al. (1999).

The particular wavelets we use are non-compact, non-orthogonal, non-discrete and can not be expressed as product of functions of x and y. This makes them quite unlike anything of current interest in the wavelet literature. However, they are closely related to analytic functions, which proves to be advantageous for reconstruction from subsets of the wavelet domain. While a proof of perfect reconstruction from multiscale edges is not available for generic signals and generic wavelets, Layson and Saloun (1998, unpublished) demonstrated that this is possible for harmonic wavelets. Their work was based on similar results obtained by Hummel and Moniot (1989) with the use of wavelets derived from the heat equation.

Practical implementations of a reconstruction algorithm are subject to computational difficulties and are far from optimal. In this paper we use the reconstruction algorithm proposed by Mallat and Zhong (1992).

Applications

Figure 1 demonstrates the quality of image reconstruction from multiscale edges. Figure 1a shows a synthetic gravity image. Figure 1b show the edges at the finest scale, while Figure 1c shows the image, reconstructed from the edges at several scales.

Next, we explore the removal of a feature from a signal. Figure 2 shows a gravity profile containing 2 anomalies. On top we show the set of multiscale edges stacked with scale coarsening in the vertical direction.

We desire to eliminate the anomaly on the left-hand-side and consequently isolate the 2 anomalies. We remove the multiscale edges branches belonging to the left side anomaly (see Figure 3).

In Figure 4 we show the result of this processing. Figure 4b contains the reconstructed profile. The anomaly on the left side has been strongly attenuated. Figure 4c shows the difference between

Figure 1. Synthetic gravity image (a), edges at the finest scale (b) and image reconstructed from the multiscale edges only (c).

the original profile and the reconstructed one, effectively isolating the anomaly. The right hand side anomaly is replaced by a minor oscillation. This shows that the right hand side anomaly is left untouched by the process.

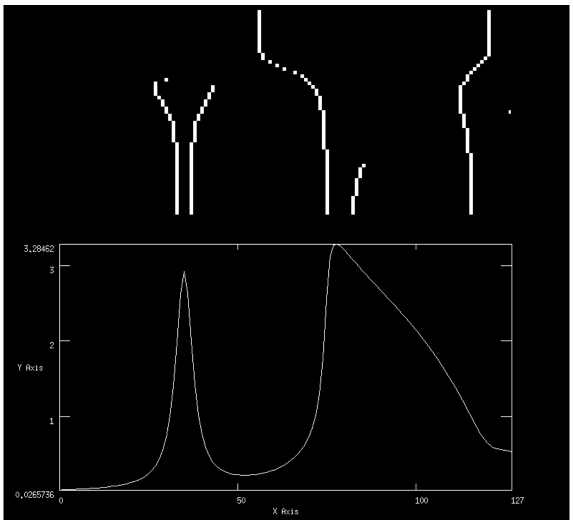

Figure 2. Gravity profile containing 2 anomalies (bottom). The set of multiscale edges (bottom).

Discussion

The reconstruction technique proposed by Mallat and Zhong (1992) works by alternate projection onto two spaces 1) the space of all functions that are wavelets transforms and 2) the space of all function whose multiscale edges coincide with the multiscale edges under analysis. That algorithm needs a modification work in our application.

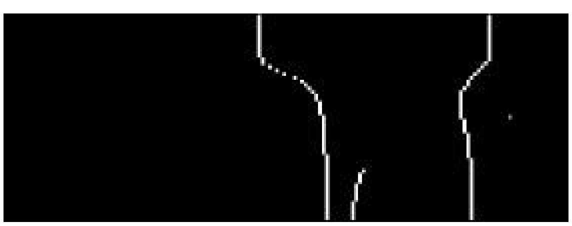

Figure 3. Edge tree from Figure 2 after removing the left hand side set.

Edges in an edge tree are related to the anomaly they are derived from, but they also contain some information about adjacent anomalies (although to a lesser extent), i.e., they are not perfectly local. When we remove some edges from the edge tree, we cause a distortion in the multiscale edge representation. Also, we are left with a set of edges that still contain information about the anomalies we are trying to eliminate. When we perform a projection into the space of wavelet transforms, this distortion results in replacing the anomaly we want to remove with an anomaly of smaller amplitude. This also means that some edges (of smaller amplitude) are (re)-generated in place of the edges we removed. To prevent we constrain the algorithm to force the location of edges *only* where the multiscale edges appeared i.e., we prevent the appearance of any unwanted edges in the edge tree. A paper with a complete description of the algorithm is in preparation.

Conclusions

We have presented an algorithm that allows the removal and isolation of features from 1D profiles. While examples have been drawn from geophysical data the same technique can work on different applications with minor modifications. One application may be in connection with a visualisation package, where multiscale edges are visualised together with the original image. Specific features can be selected and processed (either enhanced or removed) by acting on their multiscale edges in the transformed domain.

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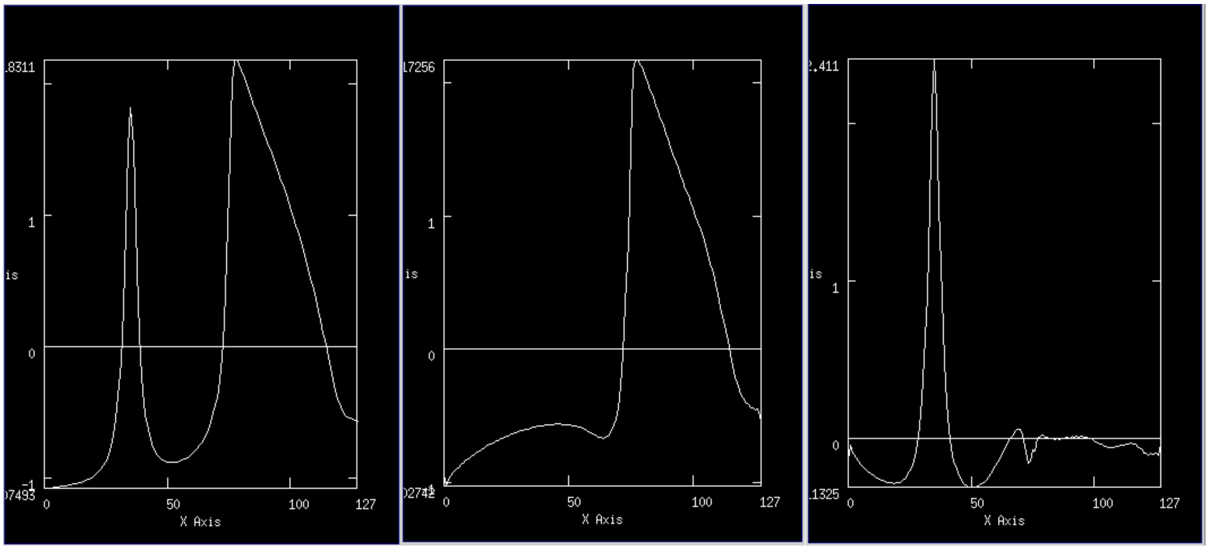

Figure 4. Original gravity profile (a). Profile after removing the left hand side anomaly (b). Difference between profile (a) and (b). The right hand anomaly remains basically untouched.

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